HEAT TRANSFER BETWEEN A STREAM OF LIQUID AND A POLYDISPERSION IN A FLUIDIZED BED

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An approximate formula is derived for the heating time of particles in a fluidized polydisperse bed.

Knowing the time in which particles will heat up to a given temperature is important in many chemical engineering processes, in drying, and in heat treatment in intermittently operating fluidization beds. While this problem can be solved easily in the case of a monodispersed material, and has been already solved with various degrees of approximation [1-4], it is difficult to solve when the solid phase of a fluidized bed contains a polydispersed material and the heat transfer particles of different sizes and at different temperatures must be taken into account. The problem is solved, to the first approximation, by reducing it to the problem of heating a monodispersion bed whose particles are of an equivalent diameter [1, 2]. Many effects related to polydispersivity and often of crucial importance remain then neglected [5, 6].

We consider a homogeneous isothermal fluidized polydispersion bed in a reactor of uniform cross section and with a given size distribution $\varphi(R)$ of spherical particles for which the following normalization applies:

$$\int_{R_1}^{R_2} \varphi(R) dR = N.$$  

Let the heat transfer between particles and the medium begin after a hot fluidizing agent or a fresh charge of cold material has been added. We will then assume that the temperature of the medium remains at this time constant and equal to the entrance temperature. The conditions under which this premise is valid follow directly from an analysis of the equations [1-3]:

$$c_G p_G v_G (t_G - t_0) = \alpha_1 4\pi R^2 hN (t_s - t_G),$$

$$c_G C_G v_G (t_G - t_0) \, dt = \varphi \rho S 4\pi R^3 hN dts.$$  

Indeed, if $\varphi 4\pi R^3 hN/c_G \rho G v_G \approx 0$ or $\varphi \rho S 4\pi R^3 hN/3c_G \rho G v_G \approx 0$, then $t_G \approx t_0$.

In subsequent calculations this condition will be assumed satisfied. In the course of heating up, particles with radius R will receive heat not only from the fluidizing agent but also from other particles at higher temperatures, while they will transmit heat to particles at lower temperatures through their contact with them.

We will assume further that particles in a fluidized bed are heated by the following mechanism, which was first mentioned in [3]. The packet of particles heated to a reference temperature $\sigma$ near the temperature of the medium comprises with it a homogeneous diluted bed containing also particles outside that packet at a lower temperature. In the course of heating, obviously, the number of elements in the fluidized packet of particles will increase.

Noting that $Bi \to 0$ for the particles and that particles with smaller diameters heat up faster, we write the equation of heat transfer between particles with radius R and the fluidized bed.

An analogous equation has been derived in [5, 6] for a throughfeed flow, but for a fluidized bed it simplifies, because the mean velocities of the various fractions must all be considered equal to zero.

Equation (3) will now be rewritten as

$$\frac{d\theta_s(R, \tau)}{d\tau} = -\frac{3\alpha(R_\tau, R)}{c_s\rho_s R} \theta_s(R, \tau).$$

(3)

Since $R_\tau$ is a function of time, we may write $\alpha(R_\tau, R) \equiv \alpha(\tau, R)$ and the solution to (4) will be

$$\ln \theta_s(R, \tau) = -\frac{3}{c_s\rho_s R} \int_{\tau_1}^{\tau} \alpha(\tau', R) d\tau' + \ln \theta_s(R, \tau_1).$$

(5)

Here $\theta_s(R, \tau)$ is the temperature of particles with the radius $R$ at the instant of time $\tau_1$, when the particles with radius $R_1$ have reached temperature $\sigma$. Temperature $\theta_s(R, \tau)$ can be found analogously to (4), if $\alpha(R_\tau, R)$ is replaced by the coefficient of heat transfer between these particles and a pure fluidizing agent — this coefficient being assumed independent of time:

$$\frac{d\ln \theta_s(R, \tau)}{d\tau} = -\frac{3\alpha_1(R)}{c_s\rho_s R}.$$

(6)

Then, with

$$\theta_s(R, 0) = 1, \quad \theta_s(R_1, \tau_1) = \sigma$$

(7)

taken into account, we have

$$\ln \theta_s(R, \tau_1) = -\frac{3\alpha_1(R)}{c_s\rho_s R} \tau_1,$$

(8)

and

$$\tau_1 = \frac{c_s\rho_s R_1}{3\alpha_1(R_1)} \ln \frac{1}{\sigma}.$$

(9)

Letting $\theta_s(R, \tau) = \sigma$ in (5), we find the time in which particles with radius $R$ heat up to temperature $\sigma$. Since the heating time $\tau$ will in this case depend on the radius $R$, one may write

$$\alpha[\tau(R), R] = \alpha(R); \quad \frac{d\tau}{dR} = \frac{dR}{dR}. \quad \frac{d\tau}{dR} \dR.$$

(10)

Then (8), (9), and (10) yield

$$\ln \sigma = -\frac{3}{c_s\rho_s R} \int_{R_1}^{R} \alpha(R') \frac{d\tau}{dR'} \dR'. \quad \frac{\alpha_1(R)}{\alpha_1(R_1)} \frac{R_1}{R} \ln \frac{1}{\sigma},$$

or

$$\int_{R_1}^{R} \alpha(R') \frac{d\tau}{dR'} \dR' = \frac{c_s\rho_s}{3} \ln \frac{1}{\sigma} \left[ R - \frac{R_1}{\alpha_1(R_1)} \alpha_1(R) \right].$$

(12)

Differentiating (12) with respect to $R$, we obtain

$$\alpha(R) \frac{d\tau}{dR} = \frac{c_s\rho_s}{3} \ln \frac{1}{\sigma} \left[ 1 - \frac{R_1}{\alpha_1(R_1)} \frac{d\alpha_1(R)}{dR} \right].$$

(13)