A method is presented for calculating the local and average heat transfer coefficients in film condensation of moving vapor of a nonmetallic fluid on a horizontal cylinder.

An analysis of the process of film condensation of moving vapor on the outer surface of a crossflow horizontal cylinder was the subject of study by a number of authors [2-9]. The solution of the problem for the case of stationary vapor is given in Nusselt's classical paper [1]. In [7, 8] the analysis of the process for the case of moving vapor is carried out with consideration of the effect of a directed mass flow through the interface on the hydrodynamics of the vapor-liquid boundary layer. However, the solution obtained in [7, 8] for the case of interest to us (for the case of the simultaneous action of friction and gravitation on the film) bears an approximate character and is in need of a more rigorous substantiation.

The analytical solution of the problem was given in [9] constructed on the physical model of the process proposed in [7, 8]. However, this solution was obtained by using certain simplifying assumptions whose legitimacy is not fully substantiated. In particular, as the authors themselves note [9], the approximation they use does not give sufficiently accurate results of the calculation for the afterbody of the cylinder. In addition, what is more important from the standpoint of use in practice, the results of calculating the average heat transfer coefficients are not given in [9] and there is no comparison with the equation of average heat transfer obtained in [7, 8]. All the foregoing also determined the need to obtain a new solution of the problem in question with the use of a more rigorous mathematical approach.

In analyzing the process (Fig. 1) the cross section of the cylinder is regarded as a regular polygon with a sufficiently large number of sides inscribed in a circle. The process of condensation on the cylinder in such a case reduces to condensation on a large number of sequentially arranged sides having different slope angles to the horizontal plane. The velocity of the vapor on the outer boundary of the vapor boundary layer and law of friction on the film surface are taken according to [7, 8]:

\[ U_\theta = 2U_\parallel \sin \varphi \]  \hspace{1cm} (1)

\[ \tau = \frac{q}{\rho U_\parallel} \]  \hspace{1cm} (2)

The resistance of the laminar film of condensate is taken as the main thermal resistance. We neglect the heat of supercooling of the condensate in comparison with the latent heat of condensation and the velocity at the interface in comparison with the vapor velocity at the outer boundary of the vapor boundary layer. In determining the geometry of the interface we neglect the thickness of the film in comparison with the diameter of the cylinder. We construct the solution for determining the local and average (over the cylinder's surface) heat transfer coefficients in the case of flow without separation at a constant surface temperature of the cylinder. To determine the average coefficient we find the average heat transfer coefficient on the first (starting from the frontal point) side and the local heat transfer coefficients on the middle generators of the other sides of the cylinder, which in connection with the sufficiently large value of the number \( n \) are equated to the average heat transfer coefficients of the sides. The average heat transfer coefficient being sought is defined as the arithmetic mean value of the indicated quantities (in view of symmetry it is sufficient to consider only on half of the cylinder). To calculate the required quantities we


©1975 Plenum Publishing Corporation, 227 West 17th Street, New York, N.Y. 10011. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, microfilming, recording or otherwise, without written permission of the publisher. A copy of this article is available from the publisher for $15.00.
Fig. 1. Scheme of analyzing the process.

Fig. 2. Comparison of the results of the given analysis with Eq. (13): 1) calculated values; 2) Eq. (13).

generalize the solution of the problem of moving vapor on a vertical plate obtained in [3] for the case of any slope in a gravitational field. For the local coefficient on the side of the cylinder with slope \( \phi_n \) with consideration of the adopted law of distribution of the vapor velocity on the outer boundary of vapor boundary layer we obtain

\[
\alpha = \sqrt{\frac{\lambda \rho U_\infty \sin \phi_n}{4 \mu x_n} \sqrt{1 + \sqrt{1 + \frac{4g x_n}{N^2 U_\infty^2 \sin \phi_n}}}},
\]

where the dimensionless complex

\[
N = \frac{\lambda \Delta T}{g l}.
\]

The average heat transfer coefficient on the first side of the cylinder

\[
\bar{z}_1 = \frac{1}{3} \sqrt{\frac{4 \lambda \rho U_\infty \sin \phi_1}{\mu}} \cdot \frac{2}{\sqrt{1 + \frac{4g l}{N^2 U_\infty^2 \sin \phi_1}}}.
\]

To find the required quantities for the other sides of the cylinder by Eq. (3), for each side we must determine the presently unknown lengths \( X_n \). For the middle generators of the sides of the cylinder these lengths are equal to

\[
X_n = \frac{1}{2}.
\]

It is obvious physically that for a correct determination of local heat transfer by Eq. (3) the length \( Z_n \) should be determined from the condition of equality of the quantities of condensate that condensed on length \( Z_n \) on one hand and on all preceding sides of the cylinder on the other. This condition is written so:

\[
\bar{z}_n (T_a - T_w) Z_n = (\alpha_1 + \alpha_2 + \ldots + \alpha_n)(T_a - T_w) l.
\]

The search for \( X_n \) according to condition (6) is done successively for each subsequent side and leads to the following calculation model:

\[
Z_1 = 0,
\]

\[
Z_n = \left[ -\left( \frac{1}{B_2^3} \frac{5C_{n-1}^2}{A_n^2 B_n^2} - \frac{C_{n-1}^4}{2A_n^4 B_n^2} \right) - \left( \frac{1}{B_n^3} \frac{5C_{n-1}^2}{A_n^2 B_n^2} - \frac{C_{n-1}^4}{2A_n^4 B_n^2} \right) \right] \left[ \frac{C_n^8}{4A_n^4 B_n^2} + \frac{3C_n^6}{A_n^4 B_n^2} + \frac{12C_n^4}{A_n^4 B_n^2} + \frac{16C_n^2}{A_n^4 B_n^2} \right]^{1/3} + \frac{2}{B_n},
\]

where

\[
A_n = \frac{1}{3} \sqrt{\frac{4 \lambda \rho U_\infty \sin \phi_n}{\mu}}.
\]

819