TURBULENT MIXING IN A SYSTEM OF PLANE NONISOThERMAL JETS

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An examination is made of turbulent flow in the main section in an infinite system of plane nonisothermal jets. The results of calculation are compared with experimental data.

We shall examine the flow formed in the basic section of the mixing zone of an infinite system of plane nonisothermal jets, discharging from nozzles of width λ/2 each. The jet exits we shall assume to be such (Fig. 1) that periodic flow occurs with period λ. In this case the streamlines passing through the middle of the nozzles will be straight lines parallel to the axis ox, and therefore it will be sufficient to examine the flow between any two streamlines separated by a distance λ, for example between lines ac and bd.

The analogous problem for the isothermal case was examined in [1].

We shall assume that \( c_p T > \sqrt{2} \), \( \Pr T = 1 \), and \( c_p = \text{const} \), and then the basic equations describing turbulent motion in the mixing zone will take the form

\[
\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y} \left( \rho e \frac{\partial u}{\partial y} \right),
\]

\[
\frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} = 0,
\]

\[
\rho u \frac{\partial T}{\partial x} + \rho v \frac{\partial T}{\partial y} = \frac{\partial}{\partial y} \left( \rho \frac{\partial T}{\partial y} \right),
\]

\[
p = \rho RT.
\]

We shall write the boundary conditions in the form

\[
\frac{\partial u}{\partial y} = \frac{\partial T}{\partial y} = v = 0 \quad \text{for} \quad y = 0, \pm \frac{\lambda}{2};
\]

\[
u = u(y), \quad T = T(y), \quad p = p_i \quad \text{for} \quad x = x_i.
\]

Transforming the first and third equations of system (1) with the help of the continuity equation, and integrating them with respect to y in the range \([-\lambda/2, \lambda/2]\), subject to conditions (2), we obtain the following integral conditions for conservation of momentum and heat:

\[
\int_{-\lambda/2}^{\lambda/2} (\rho + \rho u^2) \, dy = I_0 = \text{const},
\]

\[
\int_{-\lambda/2}^{\lambda/2} \rho u T \, dy = H_0 = \text{const}.
\]

Integrating the continuity equation over the same limits, we find an integral condition for conservation of mass:

\[
\int_{-\lambda/2}^{\lambda/2} \rho \, dy = M_0 = \text{const}.
\]

At infinity (for \( x \to \infty \)), because the mixing is complete, the stream will be uniform, i.e., it will move with constant velocity \( u_\infty \), temperature \( T_\infty \), pressure \( p_\infty \), and density \( \rho_\infty \).

For the turbulent viscosity \( \varepsilon \) we shall make use of the Prandtl hypothesis, according to which

\[
\varepsilon = K \frac{\lambda}{2} (u_{\text{max}} - u_{\text{min}}) = K \frac{\lambda}{2} \left( u(0) - u(\lambda/2) \right).
\]

Following substitution of (6) into (1), and going over to dimensionless variables according to the formulas \( x = x', y = y', u = u' u_\infty, \quad v = v' u_\infty, \quad T = T' T_\infty, \quad p = p' \rho_\infty u_\infty^2 \), and \( \rho = \rho' \rho_\infty \), system (1) may rewritten in the form (for convenience the primes in the dimensionless variables are omitted)

\[
\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y} \left( \rho e \frac{\partial u}{\partial y} \right),
\]

\[
+ K \left[ u(0) - u \left( \frac{1}{2} \right) \right] \frac{\partial}{\partial y} \left( \rho \frac{\partial u}{\partial y} \right),
\]

\[
\frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} = 0,
\]

\[
\rho u \frac{\partial T}{\partial x} + \rho v \frac{\partial T}{\partial y} = K \left[ u(0) - u \left( \frac{1}{2} \right) \right] \frac{\partial}{\partial y} \left( \rho \frac{\partial T}{\partial y} \right),
\]

\[
\rho = b \rho T.
\]

where

\[
b = RT \omega u_\infty^2.
\]

The dimensionless boundary conditions coincide in form with (2) and (2').

We shall seek expressions for \( u, T, \) and \( p \) in the form of the expansions

\[
u = 1 + \frac{a_1(y)}{x} + \frac{a_2(y)}{x^2} + \frac{a_3(y)}{x^3} + \ldots,
\]

\[
T = 1 + \frac{c_1(y)}{x} + \frac{c_2(y)}{x^2} + \frac{c_3(y)}{x^3} + \ldots,
\]

\[
p = b_0 + \frac{b_1}{x} + \frac{b_2}{x^2} + \frac{b_3}{x^3} + \ldots.
\]

Eliminating \( v \) by means of the continuity equation, transformed to the form

\[
v = -\frac{1}{\rho} \int_0^y \frac{\partial}{\partial x} (\rho u) \, dy,
\]

and the density \( \rho \) by means of the equation of state, from the first and third equations of systems (7), substituting the expansion (8) into these equations and equating coefficients of the same powers of \( x \) on the left and right sides, we obtain the following system of
equations to determine the coefficients $a_1$ and $c_1$ of the expansion

$$a'_1 + a'/K \delta_1 = -b_1/K \delta_1,$$

$$2a_2/K \delta_1 = a'_1 - a'_1,$$

$$-2a_2/K \delta_1 = a'_2 - a'_2,$$

$$-2a_2/K \delta_1 = a'_3 - a'_3,$$

$$-2a_2/K \delta_1 = a'_4 - a'_4,$$

$$-2a_2/K \delta_1 = a'_5 - a'_5.$$

Then system (10) and (11) together with conditions (13) represents a closed system of ordinary linear differential equations, which may be integrated successively.

The general solution of the first equation of system (10) has the form

$$a_1 = A_1 \cos V \sqrt{\frac{1}{K}} \delta_1 y + B_1 \sin V \sqrt{\frac{1}{K}} \delta_1 y.$$

Because of conditions (12), we must put $B_1 = 0$, $V \sqrt{\frac{1}{K}} \delta_1 = -2m$.

The solution of (16) may be rewritten in the form

$$a_1 = A_1 \cos (2m y) = q (0) \cos (2m y).$$

Satisfying the second condition of system (13), we obtain $b_1 = 0$. From equality (i8), using (17), we find

$$A_1 = a_1 (0) = -a_1 (1/2) = \frac{1}{2} a_1 (0).$$

The constant of integration $a_1 (0)$ is as yet undefined.

The solution of the first equation of system (11), satisfying condition (12), is

$$c_1 = A_1 \cos (2m y) = c_1 (0) \cos (2m y).$$

Satisfying the second condition of system (13), we obtain $b_1 = 0$. From equality (i8), using (17), we find

$$A_1 = a_1 (0) = -a_1 (1/2) = \frac{1}{2} a_1 (0).$$

The constant of integration $c_1 (0)$ is as yet undefined.

In a similar way we may integrate the remaining equations and find that

$$a_1 = A_1 \cos (2m y) = b_1.$$

The solution of the first equation of system (11), satisfying condition (12), is

$$c_1 = A_1 \cos (2m y) = c_1 (0) \cos (2m y).$$

Satisfying the second condition of system (13), we obtain $b_1 = 0$. From equality (i8), using (17), we find

$$A_1 = a_1 (0) = -a_1 (1/2) = \frac{1}{2} a_1 (0).$$

Then system (10) and (11) together with conditions (13) represents a closed system of ordinary linear differential equations, which may be integrated successively.

The general solution of the first equation of system (10) has the form

$$a_1 = A_1 \cos \sqrt{\frac{1}{K}} \delta_1 y + B_1 \sin \sqrt{\frac{1}{K}} \delta_1 y = b_1.$$

Because of conditions (12), we must put $B_1 = 0$, $\sqrt{\frac{1}{K}} \delta_1 = 2n$. The solution of (16) may be rewritten in the form

$$a_1 = A_1 \cos (2n y) = q (0) \cos (2n y).$$

Satisfying the second condition of system (13), we obtain $b_1 = 0$. From equality (i8), using (17), we find

$$A_1 = a_1 (0) = -a_1 (1/2) = \frac{1}{2} a_1 (0).$$

The constant of integration $c_1 (0)$ is as yet undefined.