A system of two differential equations of parabolic type is examined. A boundary value problem is set up and solved. A system of integrodifferential equations is obtained for determining the unknown functions. A method of reducing this to a system of ordinary Volterra integral equations is given.

Consider the system of differential equations

$$\frac{\partial U_i}{\partial t} = \sum_{k=1}^{2} a_{ik} \left( \frac{\partial^2 U_k}{\partial x^2} + \frac{\partial^2 U_k}{\partial y^2} \right) \quad (i = 1, 2).$$

We impose the following conditions on the real coefficients $a_{ik}$:

$$(a_{11} + a_{22})^2 - 4(a_{11}a_{22} - a_{12}a_{21}) = 0, \quad a_{11} + a_{22} > 0.$$  (2)

If (2) is satisfied, the roots of the characteristic equation

$$\begin{vmatrix} a_{11} - \lambda & a_{12} \\ a_{21} & a_{22} - \lambda \end{vmatrix} = 0$$

will be positive and multiple, i.e., $\lambda_1 = \lambda_2 = \lambda > 0$.

With these assumptions we shall solve the following boundary problem.

Problem: To find a solution of the system of equations (1) in the region $\Pi: t > 0, 0 < x < l, -\infty < y < +\infty$, satisfying the initial condition

$$U_i(x, y, t)|_{t=0} = 0 \quad (i = 1, 2)$$

and the boundary conditions

$$\begin{align*}
(a_1^{(1)}U_1 + a_2^{(1)}U_2)igg|_{x=0} &= \psi_1(y, t), \quad \left( \frac{\partial U_1}{\partial x} + h_1U_1 \right)igg|_{x=0} = \psi_2(y, t), \\
(a_1^{(2)}U_1 + a_2^{(2)}U_2)igg|_{x=l} &= \psi_3(y, t), \quad \left( \frac{\partial U_2}{\partial x} + h_2U_2 \right)igg|_{x=l} = \psi_4(y, t),
\end{align*}$$

where $\alpha_i, h_i (i, k = 1, 2)$ are given constants; $\psi_i(y, t) (i = 1, 2, 3, 4)$ are known continuous finite functions having continuous finite partial derivatives of sufficiently high order, and $\psi_i(y, 0) = 0$. This problem was solved in [3] for the case when the roots of the characteristic equation (2) are positive and different.

We shall seek a solution in the following form [4, 5]:

$$U_i(x, y, t) = \sum_{k=1}^{2} B_{ij}^{(1)} g^{(1)} + \omega_{ik}[x, y, t] - \sum_{k=1}^{2} B_{ij}^{(2)} g^{(2)} + \omega_{ik}[x, y, t] +$$

$$+ \sum_{k, j=1}^{2} B_{ij}^{(3)} g^{(3)} + \omega_{ik}[l - x, y, t] - \sum_{k=1}^{2} B_{ij}^{(4)} g^{(4)} + \omega_{ik}[l - x, y, t],$$

where

$$g^{(1)}(x, y, t) = \frac{1}{2\pi \lambda t} \exp \left[ -\frac{x^2 + y^2}{4\lambda t} \right], \quad g^{(2)}_{x} = \frac{\partial}{\partial x} g^{(1)}, \quad g^{(2)}_{xx} = \frac{\partial^2}{\partial x^2} g^{(1)};$$
\[ g^{(2)}(x, y, t) = \frac{x^2 + y^2}{8 \pi \lambda^2 t^{4}} \exp \left[-\frac{x^2 + y^2}{4 \lambda t}\right], \quad g_{x} = \frac{\partial}{\partial x} g^{(2)}, \quad g_{xx} = \frac{\partial^2}{\partial x^2} g^{(2)}; \]

\[ g \cdot \omega [x, y, t] = \int_{0}^{\infty} \frac{d \tau}{\tau^{3/2}} \int_{-\infty}^{\infty} g(x, y - \eta, t - \tau) \omega(\eta, \tau) d \eta. \]

The coefficients \( b_{ij} \) in (6) are defined as follows [4]:

\[
\begin{align*}
B_{11}^{1} &= a_{11}, \quad B_{12}^{1} = \lambda - a_{11}, \quad B_{21}^{1} = a_{12}, \quad B_{22}^{1} = -a_{12}, \\
B_{11}^{2} &= a_{21}, \quad B_{12}^{2} = -a_{21}, \quad B_{21}^{2} = a_{22}, \quad B_{22}^{2} = \lambda - a_{22}.
\end{align*}
\]

It is then easy to verify that the functions \( U_{i}(x, y, t) \) given by (6) satisfy (1) and (4).

The unknown functions \( \omega_{ik}(y, t) \) \( (i, k = 1, 2) \) in (6) must be defined so that functions \( U_{i}(x, y, t) \) still satisfy boundary conditions (5). For this purpose functions (6) must be substituted in (5).

We first present the following lemmas:

**Lemma 1**: If the function \( \omega(y, t) \) has a finite derivative \( \partial \omega / \partial t \) and \( \omega(y, 0) = 0 \), then

\[
\lim_{x \to 0} \frac{\partial}{\partial x^2} \int_{0}^{\infty} \frac{d \tau}{\tau^{3/2}} \int_{-\infty}^{\infty} \omega(\eta, \tau) \left[x^2 + (y - \eta)^2\right] \exp \left[-\frac{x^2 + (y - \eta)^2}{4 \lambda (t - \tau)}\right] d \eta = -\frac{1}{2 \lambda} g^{(1)}(t) \frac{\partial \omega}{\partial t}(y, t). \]

**Proof**: We first transform the integral under the derivative sign in (8) as follows:

\[
J = \int_{0}^{\infty} \frac{d \tau}{\tau^{3/2}} \int_{-\infty}^{\infty} \frac{\partial \omega(\eta, \tau)}{\partial \tau} \left[x^2 + (y - \eta)^2\right] \exp \left[-\frac{x^2 + (y - \eta)^2}{4 \lambda (t - \tau)}\right] d \eta = -\int_{0}^{\infty} d \eta \int_{-\infty}^{\infty} \frac{\partial \omega(\eta, \tau)}{\partial \tau} \left[x^2 + (y - \eta)^2\right] \exp \left[-\frac{x^2 + (y - \eta)^2}{4 \lambda (t - \tau)}\right] d \tau.
\]

Integrating the inner integral on the right side of (9) by parts, we obtain

\[
J = \frac{1}{2 \pi \lambda} \int_{-\infty}^{\infty} \frac{d \tau}{\tau^{3/2}} \frac{1}{2 \lambda} \frac{\partial \omega(\eta, \tau)}{\partial \tau} \exp \left[-\frac{x^2 + (y - \eta)^2}{4 \lambda (t - \tau)}\right] d \eta.
\]

Then

\[
\lim_{x \to 0} \frac{\partial}{\partial x^2} J = \lim_{x \to 0} \int_{0}^{\infty} \frac{d \tau}{\tau^{3/2}} \int_{-\infty}^{\infty} \frac{1}{2 \lambda} \frac{\partial \omega(\eta, \tau)}{\partial \tau} \frac{\partial^2}{\partial x^2} \exp \left[-\frac{x^2 + (y - \eta)^2}{4 \lambda (t - \tau)}\right] d \eta = 0.
\]

Since

\[
\lim_{x \to 0} \int_{0}^{\infty} \frac{d \tau}{\tau^{3/2}} \int_{-\infty}^{\infty} \frac{\partial \omega(\eta, \tau)}{\partial \tau} \frac{x^2}{4 \pi \lambda^2 (t - \tau)^2} \exp \left[-\frac{x^2 + (y - \eta)^2}{4 \lambda (t - \tau)}\right] d \eta = 0,
\]

We have

\[
\lim_{x \to 0} \frac{\partial^2}{\partial x^2} J = -\frac{1}{2 \lambda} \int_{0}^{\infty} \frac{d \tau}{\tau^{3/2}} \int_{-\infty}^{\infty} g^{(1)}(0, y - \eta, t - \tau) \frac{\partial \omega(\eta, \tau)}{\partial \tau} d \eta = 0.
\]