Formulas are presented for calculating the process of filling a vessel with a gas. These formulas were obtained by integration of the differential equations of thermodynamics of a variable mass, allowing for variation in the parameters of the incoming gas, and for heat transfer between the walls and the gas.

References [1] and [3], dealing with the calculation of gas-vessel filling processes, make the assumption that the parameters of the entering gas are constant. Change in the flow rate of the gas is taken into account only through the increase in back pressure of the vessel being filled. Actually, the filling of pneumatic systems is associated in many cases with the emptying of other vessels. Therefore, the parameters of the incoming gas are a function of time. The relevant quantities—pressure, temperature, density, and flow rate—are determined by formulas obtained in [2]. We consider the solution of the problem of a vessel being filled with a gas when the parameters of the entering gas are known functions of time.

We write the differential equation describing variation of the gas parameters in the vessel while it is being filled [7]:

\[
\frac{d\rho_2}{d\tau} = \frac{k-1}{V_2} \left( \frac{dQ_2}{d\tau} + iG_1 \right),
\]

\[
\frac{d\rho_1}{d\tau} = \frac{k-1}{G_1} \left( \frac{dQ_2}{d\tau} + iG_1 \right).
\]

The variation of flow rate* and temperature of the gas arriving in the vessel is determined by the formulas [2]

\[
\dot{G} = \frac{G_1}{G_{el}} = \frac{a}{\sqrt{\theta_2^5}} \frac{sh^5(\omega\tau + \varepsilon)}{th(\omega\tau + \varepsilon)},
\]

\[
\frac{\bar{T}}{T_{el}} = \frac{T_1}{T_{el}} = a^2 ch^3(\omega\tau + \varepsilon).
\]

The constants \(\alpha, \beta, \delta, \omega,\) and \(\varepsilon\) appearing in these expressions are given by the formulas of [8] in terms of the parameters of the supply system.

Dividing (1) by (2), and considering the familiar expression for enthalpy and Eqs. (3) and (4), we obtain the differential equation

\[
\frac{d\rho_2}{d\tau} = \frac{\alpha}{\sqrt{\theta_2^5}} \frac{G_{el}}{V_2} \frac{sh^5-1(\omega\tau + \varepsilon)ch(\omega\tau + \varepsilon)}{th(\omega\tau + \varepsilon)}.
\]

Integrating over the range 0 to \(\tau\) and from \(\rho_{el}\) to \(\rho_2 [5],\) we obtain

\[
\rho_2 = \rho_{el} + \frac{V_1}{V_2} \frac{\rho_{el}}{\sqrt{\theta_2^5}} \left[ \sqrt{\theta_2^5} - sh^5(\omega\tau + \varepsilon) \right].
\]

Now we shall determine the rate of heat supplied to the vessel being filled \(dQ_2/d\tau\) appearing in Eqs. (1) and (2). Taking the wall temperature of the vessel to be constant and equal to \(T_{el}\) (see [2]), in accordance with [6], we can write

\[
\frac{dQ_2}{d\tau} = \alpha (T_{el} - T_2) F_2.
\]

Because of the great velocity of the incoming gas, strong mixing occurs in the vessel being filled. Therefore, we can consider that the heat-transfer coefficient \(\alpha\) is given by the formula for turbulent flow [6]:

\[
\alpha = ZK_T K_t \omega^{0.5} D^{-0.2}.
\]

The function \(Z\) depends on the temperature and is proportional to the 0.8-th power of the pressure. Using an approximation based on experimental data for \(Z,\) and taking into account the temperature dependence of the correction for nonisothermicity, we can formulate an analytical expression of the form (see [4])

\[
ZK_T = \chi_2 \left( \frac{p}{T} \right)^{0.8},
\]

where the coefficient \(\chi_2\) depends on the properties of the working substance (for air \(\chi_2 = 3.53 \cdot 10^{-2} \text{kg}^{0.2} \cdot \text{sec}^{-0.6} \cdot \text{deg}^{-0.2} \cdot \text{m}^{-0.8}).\) Assuming the approximation

\[
\omega = \frac{4G_1}{\pi D^2 \rho_2},
\]

and taking account of Eq. (9), we find the value of the heat-transfer coefficient \(\alpha\) and, thus, the heat supply rate

\[
\frac{dQ_2}{d\tau} = \vartheta (T_{el} - T_2) G_1^{0.8},
\]

where

\[
\vartheta = 1.213 \chi_2 K_t R^{0.8} D^{-1.8} F_2.
\]

Now substituting this expression into (1), and taking account of Eqs. (3), (4), and (6), we obtain the differential equation

\[
\frac{d\rho_2}{d\tau} = \varphi_1 \left( \frac{sh^5 t}{th t} \right)^{0.8} + \varphi_2 \frac{sh^5 t}{th^2 t} + \varphi_3 \left( \frac{sh^5 t}{th t} \right)^{0.8} \frac{\rho_2}{(b_1 - sh^5 t)}.
\]

*We assume, for simplicity, that the pressure drop is critical in both vessels.
Fig. 1. Thermodynamic process of compression of a gas for various values of $m$: 
1) isothermal process ($m = 1$); 2) compression of the gas during filling of the vessel with $m = m(t)$; 3) adiabatic process ($m = 1.4$).

Fig. 2. Temperature of the gas (°K) in the vessel being filled as a function of time (sec).

Fig. 3. Index for the thermodynamic process of compression of the gas in the vessel being filled as a function of time (sec).