TRANSIENT MASS TRANSFER AT A PLATE AND AT THE FRONTAL STAGNATION POINT IN A LONGITUDINAL STREAM OF A NONLINEARLY VISCOUS FLUID

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Results are shown of a theoretical study concerning the transient convective mass transfer at a semi-infinitely large plate and in the frontal stagnation zone in a stream of a nonlinearly pure-viscous fluid with a power-law concentration gradient at the plate surface.

In a study of the transient convective heat and mass transfer in a stream of linearly pure-viscous fluids [1] it has been shown that at a Prandtl number Pr \( \gg 1 \) the processes of heat and mass transfer develop in a far from quasisteady mode. We will now analyze these processes under conditions of steady flow of an incompressible non-Newtonian fluid with a power-law rheological equation of state [3]:

\[
\tau_{ij} = - \rho \delta_{ij} \left[ \frac{1}{2} \left( \frac{l}{n} \right)^{\frac{1}{n-1}} \right].
\]

The original equations of the boundary layer are

\[
\frac{\partial u}{\partial x} + \frac{v}{\partial y} = \frac{V}{\partial x} + \frac{k}{\rho} \left( \frac{\partial u}{\partial y} \right)^{n-1} \left( \frac{\partial u}{\partial y^2} \right),
\]

\[
\frac{\partial c_1}{\partial t} + u \frac{\partial c_1}{\partial x} + v \frac{\partial c_1}{\partial y} = D \frac{\partial^2 c_1}{\partial y^2},
\]

where \( V = bx^i \) with \( i = 0, 1 \) for a plate and for the frontal stagnation point, respectively.

It is assumed here that the parameters in the rheological equation of state (1) as well as the physical properties do not depend on the concentration. The mass transfer is considered weak, in the sense that the concentration field of the diffusing impurity does not affect the dynamic flow characteristics. The boundary conditions for the dynamic problem are here

\[
u (x, 0) = v (x, 0) = 0; \quad u (x, \infty) = V.
\]

When the concentration gradient at the body surface varies according to a power law, then the initial and the boundary conditions for the equation of mass transfer are

\[
c_1 (x, y, 0) = c_0; \quad c_1 (x, 0, t) = 0; \quad c_1 (x, \infty, t) = c_v.
\]

For integrating the system (4), (6) we will use the so-called similarity solutions. The velocity components and the self-adjoint variables are [8]

\[
u = VF' (\eta),
\]
for a plate

\[ v = x^{-\frac{n}{1+n}} \left( \eta F' - F \right) \left( \frac{1}{1+n} \right) n \left( 1 + \frac{1}{n} \right) k \eta^2 \eta^{1/n}, \]

\[ \eta = yx^{-\frac{1}{1+n}} \left[ \frac{\eta^{2-n}}{n (1+n) k} \right] \frac{1}{1+n} = yx^{-\frac{1}{1+n}} M^{1+n}, \]

and for the frontal stagnation point

\[ v = -\frac{2bn}{1+n} x^{-\frac{n}{1+n}} \left[ \frac{2b(1-n)}{k(1+n)} \right] \frac{1}{1+n} \left( F - \frac{1-n}{2n} \eta F' \right), \]

\[ \eta = yx^{-\frac{1-n}{1+n}} \left[ \frac{2b(1-n)}{k(1+n)} \right] \frac{1}{1+n} = yx^{-\frac{1-n}{1+n}} M^{1+n}. \]

We change from physical coordinates \( x, y, t \) to coordinates \( x, \Psi, t \). Then

\[ \frac{\partial c_1}{\partial t} + u \frac{\partial c_1}{\partial x} = u \frac{\partial}{\partial \Psi} \left[ \mu \frac{\partial c_1}{\partial \Psi} \right], \]

\[ c_1 (x, \Psi, 0) = c_0; \quad c_1 (x, 0, t) = 0; \quad c_1 (x, \infty, t) = c_0, \]

where \( u = \partial \Psi / \partial y; \quad v = -\partial \Psi / \partial x \), and the flow function \( \Psi \) is defined by the relation

\[ \Psi = x^{1+n} V M^{-1+n} F (\eta) \text{ for a plate}, \]

\[ \Psi = x^{1+n} V M^{-1+n} F (\eta) \text{ for the frontal stagnation point.} \]

High values of the Prandtl number, characteristic of diffusion processes in liquids, allow us to use a linear approximation for the velocity at a wall:

\[ \frac{u}{V} = \left[ \frac{\partial (u/V)}{\partial \eta} \right]_{\eta=0} \eta = \alpha_0, \]

where the values of \( \alpha = [\partial (u/V)/\partial \eta]_{\eta=0} \) for various \( n \) have been taken from the monograph [8]. The quantities \( u, x, \) and \( \Psi \) for a plate are related as follows [9]:

\[ \eta = \left[ 2a^{-1} x^{-\frac{1}{1+n}} V^{-1} M^{-\frac{1}{1+n}} \Psi \right]^{1/2}, \]

\[ u = \left[ 2a^{-1} V M^{-\frac{1}{1+n}} \Psi \right]^{1/2}. \]

Analogously, for the frontal stagnation point we can obtain

\[ \eta = \left[ 2a^{-1} x^{-\frac{1-n}{1+n}} V^{-1} M^{-\frac{1}{1+n}} \Psi \right]^{1/2}, \]

\[ u = \left[ 2a^{-1} V M^{-\frac{1}{1+n}} \Psi \right]^{1/2}. \]

With the aid of (15), (15'), and the dimensionless quantities

\[ c = \frac{c_0 - c_1}{c_0}; \quad \tau = \frac{B^{1/3}}{x^{1/3} D} = \frac{Vt}{x} \text{ Pr}^{-1/3} A^{1/3}; \quad \omega = \frac{\psi^{1/3}}{x^2 \left( \frac{3}{4} \frac{B}{E} \right)^{1/3}}, \]

where

\[ \text{Pr} = \frac{V x D}{k} \left( \frac{V^{n-1} k x}{k} \right)^{1/2} = \text{Re} x^2 \text{Re}^{1/1+n} \]

is the universal Prandtl diffusion number,

\[ A = \left\{ \frac{a (2n+1)(n+1)}{n (n+1) \left[ 1+\frac{1}{1+n} \right] 18} \right\}; \quad E = \frac{2n + 1}{6 (n+1)}; \quad H = \frac{4 + 2n}{3 (n+1)} \]

for a plate

\[ A = \left\{ \frac{a (2n+1)}{9 (n+1) \left( \frac{2}{1+n} \right)^{1/n}} \right\}; \quad E = \frac{2 + n}{3 (1+n)}; \quad H = \frac{2n - 2}{3 (1+n)} \]