FLOW CHARACTERISTICS OF DILUTE SUSPENSIONS
WITH RIGID ELLIPSOIDAL PARTICLES IN AN
EXTERNAL FORCE FIELD

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The effect of an electric field on the rheological behavior of dilute suspensions with rigid ellipsoidal dielectric particles is analyzed.

On the basis of the structural-continuum approach, the authors in [1, 2] have derived the rheological equations of state for dilute suspensions with rigid ellipsoidal (ellipsoid of revolution) particles, considering not only their rotational Brownian movement but also the effect of external force fields on their kinematic orientation. A basic difficulty in using these equations for practical calculations is the necessity of finding the distribution function of orientation angles of the rotation axes. In several other studies [3-7] this distribution function was determined with the aid of asymptotic expansions in powers of the presumably small parameters \( K/D_r \) and \((\chi_1-\chi_2)E^2/D_rF\), these results being of little interest, however, inasmuch as the said parameters may not be small where the rheological behavior under real flow conditions of a suspension with nearly Newtonian characteristics is concerned.

In this study we will use Peterlin's method [8], which yields the distribution function also for cases other than those considered in [3-7]. We will use already known calculations for the effective viscosity and the normal stress differences as functions of the shearing rate and of the field intensity.

Let us consider a simple shear flow:

\[
\omega_z = v_z = 0, \quad v_y = Kx, \quad K = \text{const}
\]

of a suspension with rigid dielectric ellipsoids in an electric field with the intensity

\[
E_x = E \cos \alpha, \quad E_y = E \sin \alpha, \quad E_z = 0, \quad E = \text{const}.
\]

The distribution function \( F \), in spherical coordinates, satisfies the following equation [8]:

\[
\frac{\partial F}{\partial t} = D_r \Delta F - \text{div}(\omega F),
\]

(1)

The equation or orientation [1, 2] yields for our case the following components of angular velocity

\[
\omega_\phi = \dot{\phi} = \frac{K}{2} \left(1 + R \cos 2\phi\right) - \frac{\chi D_r}{2} \sin 2\phi',
\]

\[
\omega_\theta = \dot{\theta} = \frac{KR}{4} \sin 2\phi \sin 2\theta + \frac{\chi D_r}{4} (1 + \cos 2\phi') \sin 2\theta.
\]

As in the case \( E = 0 \) [8], we will also seek here the solution to Eq. (1) in the form of an asymptotic expansion in powers of the parameter \( R \) whose absolute value is not larger than unity:

\[
F(\phi', \theta) = \sum_{|m|=0} a_{nm} f_{nm}(\cos \theta) + \sum_{n=1}^N \sum_{m=1}^n \left(a_{nm} \cos 2n\phi' + b_{nm} \sin 2n\phi' \right) f_{nm}(\cos \theta),
\]

(3)

Inserting (2) and (3) into (1), we obtain the following recurrence relations for the coefficients of expansion (3) when \( \partial F/\partial t = 0 \):

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On the basis of the rheological equations of state for the given medium [1, 2] and with the aid of a computer, formulas (4) yield the effective viscosity $\mu = \mu_0 (1 + \Phi \nu)$ and the normal stress differences $T_{yy} - T_{zz} = \mu_0 \Phi K_1$ and $T_{xx} - T_{zz} = \mu_0 \Phi K_2$. The results of calculations for $\nu, f_1$, and $f_2$ as functions of $\sigma$ and $\alpha$ are shown in Fig. 1, with $\beta = 0.8$ and $a/b = 10$. The dashed line represents the case $\beta = 0$. The curves in Fig. 2 represent $\nu$ as a function of $(x_1 - x_2)E^2/D_e$ for $a/b = 10, \sigma = 0$, and $\alpha = 0$ (curve 1) or $\alpha = \pi/2$ (curve 2).