An analytical solution of the problem of the breakoff diameter of a vapor bubble and the breakoff frequency in free-convection boiling is given. It is shown that there are two bubble breakoff regimes and the boundary between them is determined.

The breakoff diameter of a vapor bubble in the boiling of liquids wetting a heating surface is usually determined from the equation proposed by Fritz

\[ d_b = 0.02 \sigma (\rho' - \rho) g^{1.2}. \]  

Equation (1) is obtained from the equilibrium condition for the bubble, which at breakoff is subjected to two forces: the upthrust and surface tension forces.

The experiments in [1] showed that formula (1) gives the breakoff diameters of vapor bubbles formed on a heating surface at relatively low heat flux densities.

An analysis of equation (1) [2, 3] and the results of several recent experimental investigations [4-6] showed that in the case of fully developed free-convection nucleate boiling the vapor-bubble breakoff diameters differ considerably from the values given by equation (1). This is attributed to the fact that formula (1) ignores the dynamic interaction of the vapor bubble growing on the heating surface with the surrounding liquid and, hence, corresponds to static vaporization conditions.

Borishanskii and Fokin [3] took into account the effect of convection currents in the boiling liquid on the detachment of the bubble from the heating surface when they determined the bubble breakoff diameter. Their expression for the bubble breakoff diameter [3] has the form

\[ d_b = \left[ 4 \cdot 10^{-4} \sigma (\rho' - \rho) g + \frac{36}{\rho^2} \left( \frac{\rho'}{\rho} \right) \left( \frac{\rho^2}{\rho'} \right)^{0.8} \left( \frac{q}{\rho \rho'} \right)^{1.0.5} \right] \]  

\[ - \frac{6}{g} \left( \frac{\rho'}{\rho} \right) \left( \frac{\rho^2}{\rho'} \right)^{0.4} \left( \frac{q}{\rho \rho'} \right)^2. \]  

Formula (2) shows that equation (1) corresponds to conditions where the heat flux density tends to zero. Another limiting solution was obtained in [7]:

\[ d_b = \frac{3 \pi^2}{(\rho' - \rho) g} \left[ \frac{1}{3} \right] \right] \left[ \frac{1}{3} \right] \left[ \frac{1}{3} \right] \left[ \frac{1}{3} \right]. \]  

In the derivation of equation (3) the surface tension force was ignored, and the resistance of the liquid was calculated from the well-known formula for the pressure resistance of a medium to the motion of spherical bodies in it:

\[ F_d = C_d \frac{\pi d^2}{4} \frac{\rho' u^2}{2}. \]
The velocity in formula (4) was determined by using the equation, given by Forster and Zuber [8], for the growth of a vapor bubble in a superheated liquid:

\[ u = \frac{d(d_b)}{dt} = 2 \frac{dR}{d\tau} = \frac{\pi a' l a^2}{R}, \]  

and the drag coefficient was taken as \( C_d = 1 \).

Equation (5) is based on the idea of a vapor bubble originating in the volume of a superheated liquid. Labuntsov et al. [9, 10] showed that this model did not correspond to nucleate boiling in which bubbles are formed on a heating surface and proposed the solution

\[ \frac{dR}{d\tau} = \beta \frac{a' l a}{R}, \]  

which takes into account the effect of the heating surface on bubble growth before breakoff. This solution agrees well with the experimental results of [10, 11].

Below we give a more general solution of the problem of bubble breakoff diameter, based on the equilibrium conditions for a bubble acted on simultaneously at breakoff by: 1) the upthrust \( F_g = \pi d^2_b (\rho - \rho') g / 6 \), 2) the surface tension force \( F_\sigma = \pi d_b \sigma(\theta) \), and, 3) the dynamic pressure force of the liquid displaced by the vapor bubble as it grows on the heating surface:

\[ F_m = \frac{d(mu)}{d\tau} = u \frac{dm}{d\tau} + m \frac{du}{d\tau}. \]  

The mass of liquid displaced by the vapor bubble during its growth is

\[ m = \frac{4\pi}{3} \rho' R^3. \]  

Determining \( u \) by using equation (6) with \( \beta = 10 \) [9]

\[ u = 2 \frac{dR}{d\tau} = 20 \frac{a' l a}{R}, \]  

we obtain from (7) and (8)

\[ F_m = \frac{1600\pi}{3} \rho' a' l a^3. \]