DECOMPOSITION OF A LAYER OF VARIABLE MASS WITH PROGRESSIVE ENERGY RELEASE

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A self-similar solution with shock wave is found for the Lagrange equations of motion of a perfect gas in which energy is released. This solution is continued by means of a difference scheme.

This paper is concerned with a gasdynamic model of the process of sublimation of a wall located in a vacuum in the presence of energy release in the wall material. The simulation of the process is divided into two parts: 1) calculation of the dispersion of the gaseous products with simultaneous energy release, and 2) calculation of the gas flow after the energy sources have ceased to act. The function Q which describes the power of the sources is so selected that the decomposition of the wall proceeds at a constant rate. In this case, the first of the two problems mentioned above is self-similar, and its solution reduces to ordinary differential equations; the second problem employs the results of the solution of the first as initial data.

1. Formulation of the problem. We have to solve the system of equations describing the motion of a perfect gas in which energy is released:

\[ \frac{\partial u}{\partial t} + \frac{\partial p}{\partial m} = 0, \quad \frac{\partial V}{\partial t} - \frac{\partial u}{\partial m} = 0, \]

\[ V \frac{\partial p}{\partial t} + \rho \frac{\partial V}{\partial t} = (\gamma - 1) \rho Q. \]  

(1)

On one side the gas is bounded by the sublimation surface; on the other its boundary is free (Fig. 1). At the free boundary

\[ p = 0, \quad \rho = 0 \quad \text{at} \quad m = 0. \]  

(2)

The conditions at the sublimation surface include the mass balance equation and the temperature condition

\[ \rho (w - u) = \rho_1 w, \quad \rho = \rho_1 T_1 \quad \text{at} \quad m = f(t). \]  

(3)

The position of the phase interface is determined from the equation

\[ \int_0^t Q [f(t), \tau] d\tau = \lambda + \epsilon_1 (T_1 - T_0) = e_1. \]  

(4)

The sublimation temperature \( T_1 \) is assumed known and independent of pressure. For many actual materials, this dependence is so weak that it can be neglected.

2. Self-similar regime. The initial instant of gas phase formation is denoted by \( t = 0 \). The point \( m = 0, \; t = 0 \) is a singular point of the solution of problem (1)-(3); at this point, the parameters of the gas are not uniquely determined, since both boundaries which figure in the conditions of the problem pass through the point. This prompts us to seek a self-similar solution.

The dependence of \( Q \) on \( m \) and \( t \) is assumed to take the form

\[ Q = C m^{-a} \rho^{a-1}, \]  

(5)

which for \( 0 < a < 1 \) expresses the decrease in the energy released with respect to mass and time. Then the determining parameters of the problem are

\[ m, \; t, \; \rho_1, \; RT_1, \; e_0, \; C, \; a, \; \gamma. \]

Apart from \( m \) and \( t \), there are only two constants with independent dimensions; consequently, for such energy release the problem has a self-similar solution.

From Eq. (4) with \( a > 0 \) it follows that the phase interface moves uniformly into the subliming material:

\[ f(t) = D t, \]

where

\[ D = (a e_1)^{-1/a} C \]

is the mass velocity of the interface. From the cor-
responding expression for C we can obtain an expression for the function Q corresponding to motion of the interface at a given velocity D:

$$Q = a \varepsilon D^g m^{-a} r^{a-1} \quad (a > 0).$$

The choice of the independent dimensionless parameter $\xi$ and the dependent parameters $U$, $v$, and $P$ is determined by the relations

$$\xi = D^{-1} m t^{-1}, \quad u = u_0 U, \quad V = V_0 v, \quad p = p_0 P,$$  

where $u_0 = V_0 D$, $p_0 = V_0 D^2$, $V_0 = V_1$.

Since $\partial t/\partial t = -(\xi/\xi) (d/d\xi)$, $\partial \xi/\partial m = (\xi/m) (d/d\xi)$, system (1) can be replaced by a system of three ordinary differential equations:

$$-\xi U' + P' = 0, \quad \xi v' + U' = 0,$$

$$v P' + \gamma P v' = -q \xi^{-a-1},$$  

where

$$q = a (\gamma - 1) \varepsilon V_1^2 D^{-2}.$$

In dimensionless form, the boundary conditions (2) and (3) become

$$P = 0, \quad v^{-1} = 0 \quad \text{at} \quad \xi = 0,$$

$$U = 1 - v, \quad P v = \tau \quad \text{at} \quad \xi = 1.$$  

Here, $\tau = RT_0^{-1} V_0^{-1}$ is the dimensionless sublimation temperature.

3. Solution in the neighborhood of $\xi = 0$. Writing system (7) in the normal form,

$$v' = q \xi^{-a-1} \Delta^{-1}, \quad P' = -q \xi^{1-a} \Delta^{-1},$$

$$U' = -q \xi^{a} v - \gamma P,$$  

where we find that the first two equations can be solved independently of the third. The point $\xi = 0$, $P = 0$, $v^{-1} = 0$ is a singular point of system (9). To construct the solution in the neighborhood of this point, we make the change of variables

$$s = \xi^2, \quad A = \xi^{1-a} v^{-1}, \quad B = \gamma P v^{-1},$$

after which the first two equations take the form

$$\frac{dA}{ds} = \frac{q A^2 + (1 - a/2) A s (B - s)}{2 s (B - s)},$$

$$\frac{dB}{ds} = \frac{q A^2 (B + v s)}{2 s (B - s)}.$$  

It is required to find a solution satisfying the conditions

$$A(0) = 0, \quad B(0) = 0.$$

Only trajectories entering the singular point at a certain tangent have physical meaning. With $A = k_1 s$, $B = l_1 s$ from (10) we obtain the following pairs of possible values for the direction numbers of the tangents $k_j, l_j$:

1) $k_1 = \left(1 + a/2\right)^{1/2} \left(1 - a/2\right)^{1/2}, \quad l_1 = \gamma \left(1 + a/2\right)^{1/2} \left(1 - a/2\right)^{1/2}.$

2) $k_2 = -k_1, \quad l_2 = l_1.$

3) any $l_3 = 0, \quad k_3 = 0.$

4. Introduction of shock wave. Trajectory (12) does not satisfy the conditions on the boundary at $\xi = 1$. Accordingly we seek a discontinuous solution of the problem with a shock wave at $\xi = \xi_*$. The states of the gas to the left and right of the wave front are linked by the Hugoniot relations. Solved for the parameters to the right of the front, these relations take the form

$$v = \frac{\xi^{-a} - a/2}{k_1}, \quad P = \frac{(1 + a/2) \xi^{1-a} \xi}{(a/2) k_1},$$

$$U = \frac{(1 + a/2) \xi^{-a}}{(a/2) k_1}. $$  

4. Calculation of flow after discontinuance of the energy release. Calculation of the flow corresponding to cessation of energy release is of interest in connection with the study of the behavior of the shock wave. This calculation consists in the numerical solution of the boundary value problem for system (1) with $Q = 0$ in the half-strip $[0 \leq m \leq D t_s, \quad t \geq t_0]$, where $t_0$ is the moment at which energy release is discontinued. The solution is obtained by means of a finite-