Solutions are obtained to linearized asymmetric heat-conduction problems for thin- and thick-walled tubes, with consideration of the radiation inside the tubes. It is shown that linearization is permissible.

The heat-conduction problem involving asymmetric heating of thick-walled tubes was solved in [1] for general nonuniform boundary conditions. If the clearance of the tube is filled with a diathermal medium, heat transfer will occur between portions of the clearance which differ in temperature. A similar phenomenon is encountered also in the design of hollow flight vehicles.

It is assumed that the degree of blackness of the wall material is unity.

Let us examine the radiation of the asymmetrically heated clearance of an infinitely long cylindrical tube (Fig. 1).

The density of the heat flux emitted by the area $dF_1$ onto $dF_2$ is equal to [2]

$$dq_{12} = \frac{\sigma T_1^4}{\pi} \cos \eta_1 \cos \eta_2 dF_2. \tag{1}$$

Considering the ratios between the segments and angles in a circular cylinder, instead of (1) we get

$$dq_{12} = \frac{\sigma T_1^4}{2\pi} \cos \frac{\pi - \theta + \varphi}{2} \cos^2 \psi d\theta d\varphi. \tag{2}$$

The expression for the flux $dq_{21}$ emitted by $dF_2$ onto the surface area $dF_1$ differs from the expression for $dq_{12}$ in that $T_1$ is replaced by $T_2$. The heat flux imparted to a unit surface area on $dF_1$ by the entire tube clearance is obtained by integrating the difference $(dq_{21} - dq_{12})$

$$q_r = \int_\phi \int_{-\pi/2}^{\pi/2} (dq_{21} - dq_{12}) d\chi =.$$  

$$\sigma \left[ \frac{1}{4} \int_\phi T^4(\psi) \sin \frac{\theta - \varphi}{2} d\theta - T^4(\varphi) \right]. \tag{3}$$

Let us compare (3) with the following linearized expression:

$$q_c = \frac{\sigma T_0^4}{4} \int_\phi T(\theta) \sin \frac{\theta - \varphi}{2} d\theta - a T(\varphi), \tag{4}$$

where the mean heat-transfer coefficient in the clearance is taken as

$$a = 4\sigma T_0^3, \quad T_0 = \frac{1}{2\pi} \int_\phi T(q) d\varphi. \tag{4a}$$

Fig. 2. Comparison of expressions (3) and (4) for $T(\varphi) = 1273 + (\Delta/2)\cos \varphi$: a) solid lines denote changes in $q_r$ (in W/m²) according to (3); points denote changes in $q_c$ according to (4); b) changes in relative error (in %) $\xi(\varphi) = 10^2[q_r(\varphi) - q_c(\varphi)]/q_r(0)$ along the tube perimeter: 1) $\Delta = 400^\circ K$, 2) $330^\circ K$, 3) $250^\circ K$, 4) $200^\circ K$, 5) $150^\circ K$, 6) $100^\circ K$, 7) $60^\circ K$, 8) $20^\circ K$. 

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The comparison between (3) and (4) for $T(q) = T_0 + (\Delta/2)\cos \phi$ is given in Fig. 2. Since the maximum error occurs for $\phi = 0$, the influence of $T_0$ and $\Delta$ on the value of $\xi(0)$ is shown in Fig. 3. From the figure, it can be seen that for values of $\Delta$ and $T_0$ frequently encountered in practice, substitution of (4) for (3) is admissible.

1. Thin-walled tube. Considering (4), the differential heat-conduction equation for a thin-walled tube has the form

$$\frac{\partial T(q, \tau)}{\partial \tau} = \frac{a}{(R_0^2)^2} \frac{\partial^2 T(q, \tau)}{\partial \phi^2} + \left[ M(\tau) + \frac{a}{c \gamma \delta} \left( 2 - \frac{1}{k} \right) \right] T(q, \tau) + \frac{a}{4c \gamma \delta} \times \left( 2 - \frac{1}{k} \right)^{2n+\phi} \times \left( \frac{2 - 1}{k} \right) \frac{T(\theta, \tau) \times \sin \frac{\theta - \phi}{2} d \theta + Q(q, \tau). \right. (5)$$

The function $T(q, \tau)$ must satisfy the condition $T(q, \tau) = T(\phi + 2\pi, \tau)$. The initial conditions are taken in the form $T(q, 0) = T_1(q)$. Functions $Q(q, \tau)$ and $T_1(q)$ must be expandable into Fourier series. The form of $Q(q, \tau)$ and $M(\tau)$ depends on the type of boundary conditions at the surfaces.

If the solution to (5) is sought in the form

$$T(q, \tau) = \sum_{m=0}^{\infty} \left[ u_m^{(1)}(\tau) \cos m \phi + u_m^{(2)}(\tau) \sin m \phi \right], \right. (6)$$

where $m = 0, 1, 2, 3, \ldots$, then the expressions for $u_m^{(1)}(\tau) \ (i = 1, 2)$ derive from ordinary differential equations which are obtained by substituting (6) into (5).

Finally we get

$$u_m^{(i)}(\tau) = \left[ \frac{am^2}{(R_0^2)^2} + \frac{a - 2 - 1/k}{c \gamma \delta} + \frac{4m^2}{1 + 4m^2} \right] \tau \times \left[ M(\tau) \right] d \tau \times \left[ \sum_{q=0}^{n} \left( Q(q, \tau) K_{m0}(q) \times \right. \right. \right.$$

$$\left. \times \exp \left[ \left( \frac{am^2}{R_0^2} \right) + \frac{a - 2 - 1/k}{c \gamma \delta} + \frac{4m^2}{1 + 4m^2} \right] \tau \right] \right. + \frac{1}{n} \int_{\theta=0}^{\pi} M(\tau) d \tau \left[ \int_{\phi=0}^{\pi} T_i(q) K_{m0}(q) d \phi \right], \right. (7)$$

where $\varepsilon_m = 2$ for $m = 0$ and $\varepsilon_m = 1$ for $m = 1, 2, 3, \ldots$, $K_{m0}(\phi) = \cos m \phi, K_{m0}(\phi) = \sin m \phi$.

2. Thick-walled tube. The differential heat conduction equations and the initial and boundary conditions at the outer surface are taken from [1]. The boundary condition at the inner surface has the form

$$a_1 \frac{\partial t(R_1, q, \tau)}{\partial \tau} = \left( b_1 + a \right) t(R_1, q, \tau) = k_1 \psi_1(q, \tau) -$$

$$- \frac{a}{4} \int_{\phi=0}^{\pi} t(R_1, 0, \tau) \sin \frac{\theta - \phi}{2} d \theta. \right. (8)$$

The solution of the problem is sought in the form

$$t(r, \varphi, \tau) = \sum_{m=0}^{\infty} \left[ c_{m1}^{(1)}(r, \tau) \cos m \varphi + c_{m2}^{(2)}(r, \tau) \sin m \varphi \right]. \right. (9)$$

where $m = 0, 1, 2, 3, \ldots$.