The polarization of a medium consisting of two-level molecules undergoing collisions with a velocity-dependent mean free time is found by a direct averaging, without the use of a kinetic equation.

1. The combined effects of Doppler broadening and collisions accompanied by a shift and quenching of levels and by a change in the collision velocities have been analyzed on the basis of the kinetic equation with a collision integral [1]. It was assumed for this analysis that the collision frequency was independent of the velocity, a valid assumption when the radiating molecules are "dissolved" in a lighter exciting gas, or when collisions with electrons play the predominant role. For molecules of a simple gas, especially for molecules "dissolved" in a heavy buffer gas, on the other hand, this dependence cannot be neglected [2].

In this paper we illustrate, for the case of the one-dimensional model, a physically graphic method based on the direct-averaging method of [3, 4]. This method takes into account the "past history" of the molecules before reaching a certain point, and it permits a natural account of the velocity dependence of the mean free time, i.e., the mean time between collisions.

2. We assume a gas of two-level molecules in a field E which depends on a single coordinate: E = E(z, t). The molecules are excited to the working levels at rates \( \lambda_1(z, t) \) and \( \lambda_2(z, t) \) (here and below the "2" and "1" refer to the upper and lower levels, respectively). Between collisions, the interaction of the molecules and the field is described in the electric-dipole approximation by the system of equations [5]

\[
\begin{align*}
\frac{\partial p_{11}(z_0, t_0, v_0, t_1, v_1, \ldots, t_K, v_K, t)}{\partial t} &= -\Gamma_1 p_{11} - \frac{1}{i\hbar} E \left( z_0 + \sum_{i=0}^{K-1} v_i (t_{i+1} - t_i) + v_K (t - t_K) \right) (p_{12} - p_{21}) ; \\
\frac{\partial p_{22}(z_0, t_0, v_0, t_1, v_1, \ldots, t_K, v_K, t)}{\partial t} &= -\Gamma_2 p_{22} + \frac{1}{i\hbar} E \left( z_0 + \sum_{i=0}^{K-1} v_i (t_{i+1} - t_i) + v_K (t - t_K) \right) (p_{21} - p_{12}) ; \\
\frac{\partial p_{12}(z_0, t_0, v_0, t_1, v_1, \ldots, t_K, v_K, t)}{\partial t} &= (-\Gamma + i\omega) p_{12} - \frac{1}{i\hbar} \times E \left( z_0 + \sum_{i=0}^{K-1} v_i (t_{i+1} - t_i) + v_K (t - t_K) \right) (p_{22} - p_{11}) ; \\
\end{align*}
\]

where \( \hat{p}(z_0, t_0, v_0, t_1, v_1, \ldots, t_K, v_K, t) \) is the density matrix of the molecule excited at point \( z_0 \) at time \( t_0 \), having a velocity \( v_0 \), and which subsequently undergoes collisions at times \( t_1, t_2, \ldots, t_K \); the velocities after the collisions are \( v_1, v_2, \ldots, v_K \), respectively; \( \Gamma_1, \Gamma_2 \), and \( \Gamma \) are the decay constants in the absence of collisions; \( E = \langle p_{12} E \rangle \); and for simplicity we assume that all the \( p_{12} \) are aligned with the field.

Our model incorporates only strong collisions [1] (collisions with uncharged molecules), and we can neglect correlations between the velocities before and after collisions as well as the shift and quenching of levels due to collisions.* Then the solution of system (1) during the time interval between collision \( \kappa - 1 \)

* We will also use this calculation method for collisions in which velocity correlation and level shift and quenching must be taken into account.
and $\kappa$ at time $t_\kappa$ becomes the initial condition for the solution in the time interval $t_\kappa - t$. Analogously, 
$\hat{\rho}(x_0, t_0, v_0, t_1, v_1, \ldots, t_{\kappa-1}, v_{\kappa-1})$ is the initial condition for $\hat{\rho}(x_0, t_0, v_0, t_1, v_1, \ldots, t_{\kappa-1}, v_{\kappa-1})$, etc.; all these density matrices are related to that $\hat{\rho}_0$ at excitation time $t_0$:

$$\hat{\rho}(x_0, t_0, v_0) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad \hat{\rho}(x_0, t_0, v_0) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

(2)

for a molecule excited to the upper or lower level. We will find it convenient to write this relation in matrix form,

$$\hat{\varphi}'(x_0, t_0, v_0, t_1, v_1, \ldots, t_{\kappa}, v_{\kappa}, t) = \hat{A}_{\kappa} \hat{\varphi}(x_0, t_0, v_0, t_1, v_1, \ldots, t_{\kappa-1}, v_{\kappa-1}, t_\kappa) = \hat{A}_\kappa \hat{A}_{\kappa-1} \ldots \hat{A}_1 \hat{A}_0 \hat{\varphi}_0,$$

(3)

where $\hat{\varphi}'$ denotes the one-column matrix

$$\hat{\varphi}' = \begin{pmatrix} \varphi_{11} \\ \varphi_{22} \\ \varphi_{12} \end{pmatrix},$$

and $A$ denotes the matrix

$$\hat{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix},$$

(4)

whose elements are given by the solution of system (1). Here the "I" shows that $A_I$ depends

on $z_0 + \sum_{l=0}^{\kappa} v_l(t_{l+1} - t_l)$, $z_0 + \sum_{l=0}^{\kappa-1} v_l(t_{l+1} - t_l)$, $t_l$, $t_{l+1}$ when $l < \kappa$; and on $z_0 + \sum_{l=0}^{\kappa-1} v_l(t_{l+1} - t_l) + v_\kappa(t - t_\kappa)$, $z_0 + \sum_{l=0}^{\kappa-1} v_l(t_{l+1} - t_l)$, $t_\kappa$, $t$, when $l = \kappa$. We can write this relation in form (3) if we solve system (1) by the method of successive approximations [5], assuming the field to be weak. We assume that the molecule is excited at $t_0$ into the upper level; then in the zeroth approximation in the field for the diagonal elements and in the first approximation for the off-diagonal elements we have, after collision $\kappa$,

$$p_{11}^{(2)} = 0;$$

$$p_{22}^{(2)}(x_0, t_0, v_0, t_1, v_1, \ldots, t_\kappa, v_\kappa, t) = p_{22}^{(2)}(x_0, t_0, v_0, t_1, v_1, \ldots, t_{\kappa-1}, v_{\kappa-1}, t_\kappa) e^{-\gamma(t - t_\kappa)};$$

$$p_{12}^{(2)}(x_0, t_0, v_0, t_1, v_1, \ldots, t_\kappa, v_\kappa, t) =$$

$$= p_{22}^{(2)}(x_0, t_0, v_0, t_1, v_1, \ldots, t_{\kappa-1}, v_{\kappa-1}, t_\kappa) e^{-\gamma(t - t_\kappa)} - \frac{1}{\hbar} p_{22}^{(1)}(x_0, t_0, v_0, t_1, v_1, \ldots, t_{\kappa-1}, v_{\kappa-1}, t_\kappa) e^{-\gamma(t - t_\kappa)}$$

$$\times \int_{t_\kappa}^{t} e^{\gamma(t - t') \gamma(t' - t_\kappa)} E \left(z_0 + \sum_{l=0}^{\kappa-1} v_l(t_{l+1} - t_l) + v_\kappa(t' - t_\kappa), t' \right) dt';$$

$$p_{22}^{(1)} = p_{22}^{(2)}.$$  

(5)

For simplicity we will use this approximation, which allows us to find the amplification line shape, the threshold conditions for generation, etc. Then the matrix $\hat{A}_\kappa$ is

$$\hat{A}_\kappa = \begin{pmatrix} 0 & 0 & e^{-\gamma(t - t_\kappa)} & 0 \\ 0 & \frac{1}{\hbar} e^{-(\gamma - \imath \omega)(t - t_\kappa)} \int_{t_\kappa}^{t} e^{\gamma(t - t') \gamma(t' - t_\kappa)} \times \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

(6)