The carrier concentration is calculated for the transient associated with a deviation in carrier concentration from equilibrium consequent on injection, accumulation, exclusion, or extraction. It is found that the duration of the transient is governed by the $rC$ of the p-n or n-n$^+$ junction when $\mu Er \gg 1$, with $rC$ equal to the time of flight. The results are compared with experiment.

Introduction

There are four distinct carrier-concentration effects in an inhomogeneous semiconductor containing a p-n junction or a junction between regions of heavy and light doping. These are injection, accumulation, extraction, and exclusion (Low's [1] terminology). Injection is the most important from the technical viewpoint, for it is the basis of many semiconductor devices; but the others control many details of the operation and also have special uses. Shockley [2] gave the first theory of injection, which has since been extensively developed; this implies that extraction occurs when the polarity of the potential difference across a p-n junction is reversed. Exclusion has been dealt with in [3, 4]; a more detailed analysis for a semiconductor of nearly Inherent conduction is given in [5]. Accumulation (reported in [6]) has been described in terms of the leak rate of a contact; a more detailed theory is given in [7]. Low [1] has shown that all four effects can be described via a unified system of equations if the concentration of minority carriers is small relative to that of the majority ones; but he neglected the fact that effects at both ends must be considered for a specimen of finite length. These effects have been considered [8] for an n-type specimen of length $L_0$ with a p-n junction at one end ($x = 0$) and an n-n$^+$ one at the other ($x = L_0$). The formulas for the steady and transient (recovery) states were derived.

My object here is to consider the transients associated with these effects when the carrier concentration deviates from equilibrium; the topic is of interest in the physics of inhomogeneous semiconductors and in uses of devices at high frequencies or under pulse conditions.

Initial Equations and Boundary Conditions

Figure 1 shows the system, the general condition being

$$p_n + p \ll n_n,$$

in which case the equations for the hole current and the deviation of hole density from equilibrium in the n-type material are [9, 10]:

$$I_p = q\nu (p + p_n) E - qD \frac{\partial p}{\partial x},$$

$$\frac{\partial p}{\partial t} = -\frac{p}{\tau} - \nu E \frac{\partial p}{\partial x} + D \frac{\partial^2 p}{\partial x^2}.$$

Here $p_n$ and $n_n$ are the equilibrium concentrations in the n-type material, $p$ is the deviation of the hole concentration from equilibrium, $I_p$ is the hole current density, $q$ is the unit charge, $\mu$ is hole mobility, $E$ is field strength [which (1) implies to be independent of the coordinates], and $D$ and $\tau$ are the diffusion coefficient and lifetime of the holes, which are related to the diffusion length $L$ by

$$L = (D\tau)^{\frac{1}{2}}.$$
Now \( \frac{\partial p}{\partial t} = 0 \) for the steady state, whereupon the boundary conditions may be put as

\[
P = p_0 \quad \text{for} \quad x = 0,
\]
\[
I_p = qvp \quad \text{for} \quad x = L_0,
\]
in which \( v \) is the rate of leak of holes into the n\textsuperscript{+} region, which can be taken as independent of \( p \) if (1) is obeyed. Then the solution to (3) for \( p \) along the specimen in the steady state is

\[
p = p_0 e^{\frac{\mu E}{2L^2} x} \left[ e^{-g x} + 2ae^{-gL_0} \sinh gx \right],
\]
with

\[
a = \frac{Dg + \frac{\mu E}{2} - v + \mu E \frac{p_n}{p_0} e^{-\frac{\mu E L_0}{2L^2} + gL_0}}{2Dg \cosh gL_0 - \mu E \sinh gL_0 + 2v \sinh gL_0}
\]
and

\[
g = \sqrt{\left(\frac{\frac{\mu E}{2L^2}\right)^2 + \frac{1}{L^2}}.
\]

To solve (3) for the transient state we need other boundary conditions; the second condition in (5) still applies, but the first must be replaced, because \( p \) for \( x = 0 \) does not reach its steady value instantly. This boundary concentration \( p(0, t) \) is found as a function of time by means of the equivalent circuit of Fig. 2, in which \( R \) is the resistance of the n-type region, \( r \) and \( C \) being the resistance and capacitance of the p-n junction. The resistance of the p-n junction is negligibly small, and the potential drop across it may be neglected (\( v = \text{constant} \)). Figure 2 shows that a constant voltage \( V \) applied to the specimen gives rise to a voltage across the p-n junction that tends to the steady value \( V_K = V/(1 + R/r) \) in accordance with

\[
V_n(t) = V_K \left(1 - e^{-t/t_0}\right),
\]

\[
t_0 = \frac{RrC}{r + R}.
\]

If a Boltzmann distribution applies,

\[
p(0, t) = p_n \left[ e^{\frac{qV_K(t)}{kT}} - 1 \right]
\]
and, substituting for \( V_K \) from (9), we have

\[
p(0, t) = p_n \left[ \left( e^{\frac{qV_K}{kT}} \right)^{1-e^{-t/t_0}} - 1 \right].
\]
But \( p_0 = p_n \left( e^{qV_K/kT} - 1 \right) \), so (11) may be put as

\[
p(0, t) = p_n \left[ \left( \frac{p_0 + p_n}{p_n} \right)^{1-e^{-t/t_0}} - 1 \right].
\]

Then \( p(x, t) \) is to be found by solving (3) subject to (12) for \( x = 0 \) and to the second condition of (5) for \( x = L_0 \).

**Diffusion Capacitance and Differential Resistance of the p-n Junction**

The diffusion capacitance of a p-n junction is [2, 9] far greater than the inherent capacitance, so the \( C \) of (10) must be taken as the diffusion capacitance, which we now calculate for the distribution of (6). We have

\[
Q_p = q \int_0^{L_0} p(x) dx = q p_0 \left[ a_1 + \frac{Dg + \frac{\mu E}{2} - v}{h} (a_2 - a_1) \right] +
\]
\[
\quad + q p_n \frac{\mu E e^{-\frac{\mu E L_0}{2L^2} + gL_0}}{h} (a_2 - a_1).
\]