CALCULATION OF RADIATIVE HEAT TRANSMISSION THROUGH DISPERSIVE MEDIA

L. A. Konyukh and F. B. Yurevich

A method is outlined for solving the equation of energy radiation and also for determining the thermal radiation flux in an emitting, an absorbing, and an anisotropically dispersing medium. Values of radiation flux calculated here agree closely with data published in the technical literature.

Calculating the radiation in a gaseous medium with a slight admixture of solid particles, liquid droplets, or opaque gases is worthwhile for the evaluation of many high-temperature processes. In order to calculate the thermal radiation flux in such contaminated media, it is necessary to solve the integro-differential equation of heat radiation. Under conditions of local thermodynamic equilibrium and with the intensity of monochromatic radiation independent of the azimuth angle \( \varphi \), this equation for a plane layer of an emitting, an absorbing, and a dispersing medium is

\[
\frac{\partial I (\tau, \mu)}{\partial \tau} + I (\tau, \mu) = (1 - \omega_o) I_b (\tau) + \frac{\omega_o}{2} \int P (\mu', \mu') I (\tau, \mu') d\mu',
\]

where function \( P (\mu, \mu') \) has been obtained from the dispersion indicatrix \( P (\theta) = \sum a_i P_i (\cos \theta) \) at the angle subtending the layer, according to the law of cosines in spherical trigonometry

\[
\cos \theta = \mu \mu' + (1 - \mu^2)^{1/2} [1 - (\mu')^2]^{1/2} \cos (\varphi - \varphi')
\]

and by subsequent integration with respect to \( (\varphi - \varphi') \) [2]:

\[
P (\mu, \mu') = \sum_{i=0}^{1} a_i P_i (\mu) P_i (\mu').
\]

We will assume that the plane layer of dispersing material is bounded by plane-parallel diffusively radiating surfaces. The boundary conditions at these surfaces will then be [3]:

\[
I (0, \mu)|_{\mu > 0} = \varepsilon_1 I_b (0) + \frac{1}{\mu} \int_{0}^{1} I (0, \mu)|_{\mu < 0} d\mu,
\]

\[
I (\tau_b, \mu)|_{\mu < 0} = \varepsilon_2 I_b (\tau_b) + \frac{1}{\mu} \int_{0}^{1} I (\tau_b, \mu)|_{\mu > 0} d\mu.
\]

Since the radiation intensity in problems of this kind is discontinuous at \( \mu = 0 \), hence it is worthwhile to split the problem into \( I^+ (\tau, \mu) \) with \( 0 < \mu < 1 \) and \( I^- (\tau, \mu) \) with \( -1 < \mu < 0 \), and then, following the Eavon method [1], to use the Legendre expansion of \( I^+ (\tau, \mu) \) into polynomials \( P_n (2\mu - 1) \) and of \( I^- (\tau, \mu) \) into polynomials \( P_n (2\mu + 1) \):

\[
I^+ (\tau, \mu) = \frac{1}{4\pi} \sum_{i=0}^{\infty} (2i + 1) I_i^+ (\tau) P_i (2\mu - 1),
\]

\[
I^- (\tau, \mu) = \frac{1}{4\pi} \sum_{i=0}^{\infty} (2i + 1) I_i^- (\tau) P_i (2\mu + 1).
\]
\[ I^\tau (\tau, \mu) = \frac{1}{4\pi} \sum_{i=0}^{\infty} (2i+1) I_i^\tau (\tau) P_i (2\mu + 1) \]

where

\[ I_i^\tau (\tau) = 2\pi \int_{-1}^{1} I^\mu \left( \tau, \frac{\mu + 1}{2} \right) P_i (\mu) d\mu, \quad I_i^- (\tau) = 2\pi \int_{-1}^{1} I^\mu \left( \tau, \frac{\mu - 1}{2} \right) P_i (\mu) d\mu. \]

In these expansions (4) we will retain only \( N \) terms, and this number \( N \) will determine the degree of approximation to which the radiation equation is solved by that method.

Inserting (4) into (1), we obtain two equations for \( I^\tau+ (\tau, \mu) \) and \( I^- (\tau, \mu) \) (here and henceforth we imply summation over \( i \))

\[ (2i+1)P_i (2\mu - 1) \frac{d I_i^\tau (\tau)}{d\tau} + (2i + 1) P_i (2\mu - 1) I_i^\tau (\tau) = 4\pi (1 - \omega_0) I_0 (\tau) \]

\[ + \frac{\alpha_{nk}}{2} \left[ (2i+1) P_i (2\mu - 1) P (\mu, \mu') d\mu' + (2i+1) I_i^\tau (\tau) \times \right. \]

\[ \times \left. \int_{-1}^{1} P_i (2\mu + 1) P (\mu, \mu') d\mu' \right], \]

\[ (2i+1)P_i (2\mu + 1) \frac{d I_i^- (\tau)}{d\tau} + (2i + 1) P_i (2\mu - 1) I_i^- (\tau) = 4\pi (1 - \omega_0) I_0 (\tau) \]

\[ + \frac{\alpha_{nk}}{2} \left[ (2i+1) P_i (2\mu - 1) P (\mu, \mu') d\mu' + (2i+1) I_i^- (\tau) \times \right. \]

\[ \times \left. \int_{-1}^{1} P_i (2\mu + 1) P (\mu, \mu') d\mu' \right]. \]

We next multiply equations (5) by \((2k + 1)P_k (2\mu - 1)\) and by \((2k + 1)P_k (2\mu + 1)\) respectively, then integrate with respect to \( \mu \) the first equation from 0 to 1 and the second equation from \(-1\) to 0. Letting \( k = 0, 1, 2, \ldots \), we obtain a system of \( 2(N + 1) \) ordinary differential equations in functions \( I^\tau_i (\tau) \) and \( I^- (\tau) \)

\[ \alpha_{ik} \frac{d I_i^\tau (\tau)}{d\tau} + \beta_{ik} I_i (\tau) = \frac{\alpha_{ik}}{2} \left( I_i^\tau (\tau) \gamma_{i+k} + I_i^- (\tau) \gamma_{-i-k} \right) + 4\pi (1 - \omega_0) I_{ik} (\tau), \]

\[ (-1)^{i+k+1} \alpha_{ik} \frac{d I_i^- (\tau)}{d\tau} + \beta_{ik} I_i (\tau) = \frac{\alpha_{ik}}{2} \left( I_i^\tau (\tau) \gamma_{i+k} + I_i^- (\tau) \gamma_{-i-k} \right) + \]

\[ + 4\pi (1 - \omega_0) I_{ik} (\tau), \]

where

\[ \alpha_{ik} = (2i+1)(2k+1) \int_{0}^{1} \mu P_k (2\mu - 1) P_i (2\mu - 1) d\mu \]

\[ = \begin{cases} 
\frac{i}{2}, & k = i+1, \\
\frac{2i+1}{2}, & k = i, \\
\frac{i+1}{2}, & k = i-1, \\
0, & k < i-1, k > i+1
\end{cases} \]

and

\[ \beta_{ik} = (2i+1)(2k+1) \int_{0}^{1} P_k (2\mu - 1) P_i (2\mu - 1) d\mu \]

\[ = (2i+1)(2k+1) \int_{-1}^{1} P_k (2\mu + 1) P_i (2\mu + 1) d\mu \]

\[ = \begin{cases} 
0, & i \neq k, \\
2i+1, & i = k
\end{cases} \]