Bianchi Type-V Perfect Fluid Models

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A study of the nontilted diagonal Bianchi type-V perfect fluid models is undertaken in the nonlocally rotationally symmetric case. It is shown that in such a model, the singularity is necessarily velocity dominated and the upper bound for the ratio of the shear to the expansion is \(1/\sqrt{3}\). A class of such exact models is also presented; the structure of the singularity and the effect of the anisotropy on the scale are discussed. Lastly, an attempt is made to solve exactly the equations of motion for the coordinates of the null geodesics.

1. INTRODUCTION

It has been both experimentally and theoretically established that the present universe is both spatially homogeneous and isotropic and therefore can be well described by a Friedmann–Robertson–Walker (FRW) model [1, 2]. But a FRW model has the disadvantage to being unstable near the singularity [3] and therefore it fails to describe an early universe. So the most appropriate model of the universe has to be necessarily inhomogeneous and anisotropic near the singularity. But for their relative mathematical simplicity, the study of the anisotropic spatially homogeneous Bianchi models is undertaken to understand the universe at its early stage of evolution. Of the nine types of Bianchi models, the only types which can tend toward isotropy at arbitrarily large times and hence permit the formation of galaxies and the development of intelligent life are types I, V, VII\(_0\), and VII\(_h\) [4]. It may be recalled that the types I and VII\(_0\) represent the generalized flat FRW models, whereas the types V and VII\(_h\)
represent the generalized open FRW models. Between the flat and open models, the study of the latter type is considered physically more relevant because the "hot spots" can occur only in these models and the positive detection of a hot spot would unequivocally demonstrate that our universe is open [5]. In view of this, the study of Bianchi type V and VII\(_h\) models assumes considerable importance in relativistic cosmology.

In literature, we find that the type-V models, both tilted and nontilted, are moderately well studied [6–26]. But our objective in the present paper is to revive the study on the nontilted type-V perfect fluid models. It may be recalled that though Ellis and MacCallum [12] have suggested the possibility of finding the exact nontilted perfect fluid solutions in terms of elliptic integrals, they have never given the exact relation between the average length \(l\) and the proper time \(t\). Therefore, in subsequent studies [27], we find the study of the quantitative aspect of these models to be limited and less precise.

In the present paper, to augment the scope of the quantitative study of these models, we lay stress on finding the exact solutions of the field equations. In Section 2, we have written down the field equations and obtained the equation of evolution for the nontilted type-V diagonal perfect fluid models in the nonlocally rotationally symmetric case. In Section 3, the general features of such models are studied. In Section 4, the equation of evolution is integrated in certain specific cases and the exact solutions of the field equations are presented. In Section 5, these solutions are studied; the structure of the singularity and the effect of the anisotropy are determined. In Section 6, an attempt is made to solve exactly the equations of motion for the coordinates of the null geodesics.

2. FIELD EQUATIONS

In orthonormal synchronous basis the energy–momentum tensor of the perfect fluid is given as

\[
T_{\mu\nu} = (P + p) u_\mu u_\nu + \eta_{\mu\nu} p
\]  

We assume the fluid to be comoving, i.e., \(u^2 = \delta_{\nu\nu}\). The field equations corresponding to the type-V diagonal metric [28] characterizing perfect fluid distribution are

\[
3\dot{\Omega} - 3\dot{\Omega}^2 - 6\dot{\beta}_+^2 - 6\dot{\beta}_-^2 = (4\pi(P + 3p))
\]

\[
6\dot{\beta}_+ e^{\Omega + 2\beta_+} = 0
\]

\[
-\dot{\Omega} + \dot{\beta}_+ + \sqrt{3} \dot{\beta}_+ + 3\dot{\Omega}^2 - 3\dot{\Omega}(\dot{\beta}_+ + \sqrt{3} \dot{\beta}_-) - 2e^{2(\Omega + 2\beta_+)} = 4\pi(P - p)
\]

\[
-\dot{\Omega} + \dot{\beta}_- - \sqrt{3} \dot{\beta}_- + 3\dot{\Omega}^2 - 3\dot{\Omega}(\dot{\beta}_- - \sqrt{3} \dot{\beta}_-) - 2e^{2(\Omega + 2\beta_+)} = 4\pi(P - p)
\]