An engineering method is proposed for determining the initiation of turbulence in a subsonic \((M \leq 1)\) laminar boundary layer with a heat supply in the presence of a pressure gradient and injection.

A large number of theoretical and experimental works have by now been devoted to the question of the transition of a laminar boundary layer into a turbulent boundary layer. However, this phenomenon (transition from laminar to turbulent boundary layer) is not amenable to rational explanation in every sense. What is needed is a methodological approach which considers both theoretical and empirical aspects of the phenomenon. Possible elements of such an approach, presented below, permit consideration of some of these aspects.

To determine the moment of loss of stability of the laminar boundary layer on the body under consideration, it is necessary to have estimates of the velocity profiles of this layer along the generatrix of the body. However, such estimates are often lacking, or obtaining them proves to be a very complex task. Thus, the stability of an incompressible laminar boundary layer is often determined by using the approximate velocity profile of K. Pohlhausen [1], which adequately describes the solutions of the equation of an incompressible boundary layer in the presence of a pressure gradient:

\[
\frac{u}{u_e} = 2\eta - 2\eta^2 + \eta^3 + \frac{\Lambda}{6} (\eta - 3\eta^2 + 3\eta^3 - \eta^4).
\] (1)

The point of loss of stability of the laminar boundary layer (the beginning of turbulence) depends heavily on the first form parameter \(\Lambda\). The dependences of \(Re\) and \(Re_t\) are given in several works. For example, \(Re = f(\Lambda)\) for an incompressible boundary layer is given in [1], while \(Re_t = f(\Lambda)\) for a subsonic compressible boundary layer is given in [2]. For rapid determination of the point of loss of stability of a laminar boundary layer with simultaneous injection and pressure gradient, propose that \(\Lambda_{ef}\) be introduced as follows.

We differentiate Eq. (1) with respect to \(\eta\):

\[
\frac{u}{u_e} = 2 - 6\eta^2 + 4\eta^3 + \frac{\Lambda}{6} (1 - 6\eta + 9\eta^2 - 4\eta^3).
\] (2)

At the wall (\(\eta = 0\)) we have

\[
\frac{u_{\text{wall}}}{u_e} = 2 + \frac{\Lambda}{6},
\] (3)

where

\[
u_{\text{wall}} = \nu_{\text{wall}} \delta.
\] (4)

With injection, \(u_{\text{wall}}\) decreases. This decrease is accounted for by multiplying \(u_{\text{wall}}\) in the absence of injection by the coefficient \(\Psi\) [3]. We then obtain

\[
u_{\text{wall}} = u_{\text{wall}} \delta \Psi = \nu_{\text{wall}} \Psi,
\] (5)

where \(u_{\text{wall}}\) is the derivative of velocity with respect to \(\eta\) in the absence of injection. There is a great deal of published data on the dependence of \(\Psi\) on the amount of injection, the composition of the injected gas, and the pressure gradient (see, e.g., [1, 3, 4]).

We substitute the value of \(u_{\text{wall}}\) from Eq. (5) into Eq. (3)

\[
\frac{\Psi u_{\text{wall}}}{u_e} = 2 + \frac{\Lambda}{6}.
\] (6)

It follows from Eq. (6) that

\[
\Lambda = \Lambda_{ef} = \left(\frac{u_{\text{wall}} \Psi}{u_e} - 2\right) 6,
\] (7)

where \(\Lambda_{ef}\) is an expression for \(\Lambda\) which considers the simultaneous effect of injection and a pressure gradient. When \(\Psi = 1.0\) (no injection):

\[
\Lambda = \Lambda_{ef} = -\frac{dp}{dx} \frac{\delta^2}{\mu u_e};
\] (8)

in this case

\[
\frac{u_{\text{wall}}}{u_e} = \frac{1}{6} \frac{dp}{dx} \frac{\delta^2}{\mu u_e}.
\] (9)

We insert \(u_{\text{wall}}/u_e\) from Eq. (9) into Eq. (7)

\[
\Lambda_{ef} = \left(-\frac{dp}{dx} \frac{\delta^2}{\mu u_e} + 12\right) \Psi - 12.
\] (10)

Using the dependence of \(Re\) on \(\Lambda\) from [1], we calculated \(Re\) for a plate \((dp/dx = 0)\) with different values of suction (or negative injection). Here, instead of \(\Lambda\) in the function \(Re = f(\Lambda)\), we inserted the value of \(\Lambda_{ef}\). Comparison of our estimates of \(Re\) with results calculated by A. Ulrich (results presented in [1]) for exact velocity profiles in accordance with stability theory showed satisfactory agreement within a fairly broad range of suction. For \(\xi = 0, 0.005, 0.02,\) and \(0.08, Re_{\text{cr}}\) was equal to 575, 1120, 1820, and 3940 according to Ulrich and 575, 1100, 1850, and 4780 according to our results.

Equation (10) pertains to the boundary layer of an incompressible liquid.