AXISYMMETRIC IMPACT OF A SHELL ON AN ELASTIC HALFSPACE

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FORMULATION OF THE PROBLEM

A thin elastic spherical shell moving perpendicular to the surface of an elastic halfspace \( z \geq 0 \) reaches this surface at time \( t = 0 \). The shell is associated with the moving spherical coordinate system \( r'\varphi'\theta' \), and the halfspace is associated with the motionless cylindrical coordinate system \( r\varphi z \). The shell enters the elastic medium at speed \( v_\varphi(t) \) (\( 0 \leq t \leq T \)); the initial penetration rate \( v_\varphi(0) \); \( T \) is the duration of the interaction between the shell and the halfspace. The shell thickness \( h \) is considerably less than the radius \( R \) of the median shell surface (\( h/R \leq 0.05 \)). We introduce the dimensionless variables

\[
\begin{align*}
\tau &= \frac{C_s t}{R}; \quad \zeta = \frac{r}{R}; \quad \varphi = \frac{\varphi}{\varphi_0}; \quad \theta = \frac{\theta}{\theta_0}; \quad \nu_\varphi = \frac{\nu_\varphi}{\nu_\varphi(0)}; \quad \nu_\theta = \frac{\nu_\theta}{\nu_\theta(0)}; \quad \varphi_0 = \frac{\varphi_0}{\varphi_0(0)}; \quad \theta_0 = \frac{\theta_0}{\theta_0(0)}; \quad \beta_{\varphi} = \frac{\beta_{\varphi}}{\beta_{\varphi}(0)}; \quad \beta_{\theta} = \frac{\beta_{\theta}}{\beta_{\theta}(0)}; \\
\nu_\varphi &= \frac{v_\varphi}{v_\varphi(t)}; \quad M = \frac{M}{\rho R^2}; \quad \alpha^2 = \left( K + \frac{4}{3} \mu \right)/K; \quad \beta^2 = \mu/K; \quad C_s^2 = \left( K + \frac{4}{3} \mu \right)/\rho; \quad C_p^2 = \frac{K}{\rho},
\end{align*}
\]

(1)

where \( \rho, \mu, K, C_p, \) and \( C_s \) are the density, shear modulus, bulk deformation modulus, and wave velocity in the elastic halfspace; \( E_0, \nu_0, \rho_0, M \) are the Young’s modulus, Poisson’s ratio, density, and linear mass of the shell. Everywhere below, the primes are omitted.

The motion of a spherical shell is described by the basic system of dynamic equations for thin elastic shells, based on the Kirchhoff—Love hypotheses

\[
\begin{align*}
\frac{\partial^2 u_\varphi}{\partial \varphi^2} + \text{ctg} \theta \frac{\partial^2 u_\varphi}{\partial \theta^2} - (\nu_\varphi + \text{ctg}^2 \theta) u_\varphi - (1 + \nu_\varphi) \left( \frac{\partial w_\varphi}{\partial \varphi} + \alpha_u \left( \frac{\partial^2 w_\varphi}{\partial \varphi^2} + \frac{\partial^2 u_\varphi}{\partial \theta^2} \right) \right) &+ \text{ctg} \theta \left( \frac{\partial^2 w_\varphi}{\partial \theta^2} + \frac{\partial^2 u_\varphi}{\partial \theta^2} \right) - (\nu_\varphi + \text{ctg}^2 \theta) \left( \frac{\partial w_\varphi}{\partial \theta} + u_\varphi \right) = \beta_1 \frac{\partial^2 u_\varphi}{\partial \varphi^2}, \\
(1 + \nu_\varphi) \left[ \frac{\partial u_\varphi}{\partial \varphi} + \text{ctg} \theta \nu_\theta - 2 w_\varphi \right] &- \frac{\partial u_\varphi}{\partial \theta^2} \left( \frac{\partial w_\varphi}{\partial \varphi} + \frac{\partial^2 u_\varphi}{\partial \theta^2} \right) - 2 \text{ctg} \theta \left( \frac{\partial^2 w_\varphi}{\partial \theta^2} + \frac{\partial^2 u_\varphi}{\partial \theta^2} \right) - \left( 1 + \text{ctg}^2 \theta + \nu_\varphi \right) \left( \frac{\partial w_\varphi}{\partial \theta} + u_\varphi \right) = \beta_2 \frac{\partial^2 w_\varphi}{\partial \theta^2} - \beta_2 p,
\end{align*}
\]

(2)

where \( \alpha_u = h^2/(12 R^2); \quad \beta_1 = (1 - \nu_0^2) \rho_0 K/(E_0 \nu_0); \quad \beta_2 = (1 - \nu_0^2) K R/(E_0 h); \quad u_\varphi, w_\varphi \) are the displacements of the points of the shell median surface.

The equations of motion of the elastic halfspace are written in the form [5]

$$\Delta \varphi = \frac{1}{\alpha^2} \frac{\partial^2 \varphi}{\partial t^2}; \quad \Delta \psi = \frac{1}{\beta r} \frac{\partial^2 \psi}{\partial t^2}; \quad \Delta = \frac{\partial^2}{\partial r \partial r} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \varphi \partial \varphi} + \frac{\partial^2}{\partial z \partial z}. \quad (3)$$

If the shear modulus $\mu$ is zero, the equations of motion of the elastic medium reduce to the acoustics equations. Taking account of Eq. (1), the physical quantities are expressed in terms of the wave potentials as follows

$$u_z = \frac{\partial \varphi}{\partial z} + \frac{\partial \psi}{\partial t} + \psi; \quad \sigma_{zz} = \left( 1 - 2 \beta^2 \right) \frac{\partial^2 \varphi}{\partial t^2} + 2 \beta^2 \left( \frac{\partial^2 \varphi}{\partial r \partial r} + \frac{\partial^2 \psi}{\partial r \partial \varphi} + \frac{1}{r} \frac{\partial \psi}{\partial r} \right);$$

$$\sigma_{\varphi z} = 2 \beta^2 \left( \frac{\partial^2 \psi}{\partial r \partial z} + \frac{1}{2 \beta^2} \frac{\partial^2 \varphi}{\partial z^2} - \frac{\partial^2 \psi}{\partial z^2} \right); \quad (4)$$

$u_z$ is the vertical component of the displacement vector; $\sigma_{zz}$, $\sigma_{\varphi z}$ are the stress–tensor components.

The approach of [5] is adopted; in this approach, it is assumed that the linear coordinates along the halfspace and shell surfaces are the same in the early stages of penetration and consequently

$$r \approx \theta; \quad \text{ctg} \theta \approx \frac{1}{\theta}. \quad (5)$$

In the contact region, taking account of Eq. (5), the relations between the displacements $u_z$, $w_0$ and the pressure $p$ take the form

$$u_z \bigg|_{z=0} = w_r(t) - 1 + \sqrt{1 - r^2} - w_0(t, r), \quad r < r^*; \quad w_r(t) = \int_0^t v_r(\tau) d\tau; \quad (6)$$