The oscillatory pattern of variation in the $\sigma_{12\text{max}}(t)$ function is associated with the diffraction of waves on the unrestrained cutouts and their circulation along the transverse coordinate $x_2$. In contrast to points 1-3, the evolution of stresses at point 4, which is located on the perimeter of the unrestrained opening, where $\sigma_{12\text{max}} = \sigma_{11}$, does not lead to their repeated zeroing in the observation interval.

LITERATURE CITED


EVALUATION OF THE BRAKING TIME AND PATH OF A FOUR-WHEEL VEHICLE

Ya. K. Lyubimtsev, V. S. Metrikin, and N. A. Pufaev

The problem of the braking of a four-wheel vehicle moving along a rough plane is examined. Equations of motion of the vehicle are derived with allowance for arresting devices and conditions for the existence of various types of motion (with and without sliding of the wheels). Analytical relationships for estimating the braking and values of variables defining the position of the vehicle over time are derived using the method of Lyapunov functions (MLF) and methods of comparison.

1. Problem Statement. Let us represent a four-wheel vehicle in the form of a rigid body, which is symmetric with respect to its own longitudinal median plane, the latter being established using four elastic shock absorbers on rigid disks of radius $r_0$, which interact with the support plane and with the brake blocks via dry-friction forces conforming to the Coulomb-Amonton law (Fig. 1). It is assumed that the vehicle's suspension is dependent, and the steering system is rigid and free of play and sluggishness. For these assumptions, the problem of the vehicle's braking is reduced to study of the dynamics of a system with unrestrained kinematic couplings.

Let us introduce the stationary coordinate system $Oxyz$, the plane of which coincides with the profile of the road and the following moving coordinate systems: $O_2x_2y_2z_2$, which is affixed to the body so that the $x_2$ and $y_2$ axes lie in the plane passing through the upper points of shock-absorber attachment, and the axis passes through the center of mass of the body, and $O_1x_1y_1z_1$, the origin of coordinates of which coincides with projections of point $O_2$ on the $(x, y)$ plane (Fig. 1). The Cartesian coordinates $x$, $y$, $z$ of point $O_2$, the gallop angle $\alpha$, the angle of lateral tilt $\beta$ of the body, the angle $\theta$ between the longitudinal axis of the vehicle and the $x$ axis, the angle $\psi$ of rotation of the plane of the front right wheel, which will hereinafter be considered constant, and the turn angles $\varphi_i (i = 1, 4)$ of the wheels during their natural rotation.

The angle $\psi_1$ of rotation of the plane of the left front wheel with the condition that all wheels are rolling without lateral slip is determined from the relationship (Fig. 2)

$$\cot \psi_1 = \cot \psi - \lambda (\lambda = 2b/a).$$  \hspace{1cm} (1.1)
where $b$ is the length of half the axle, $a = a_1 + a_2$ is the vehicle base, and $a_i$ is the distance from point $O_i$ to the projection of the front ($i = 1$) and rear ($i = 2$) axles onto the plane of the road.

In deriving the vehicle's equations of motion, we can use equations in the quasi-coordinates $\eta_j$ ($j = 1, 2, \ldots, 11$) [2]

\[
\frac{d}{dt} \frac{dL^*}{d\eta_i} - \frac{\partial L^*}{\partial \eta_i} + \gamma_{\eta} \frac{\partial L^*}{\partial \eta_i} \eta_i = \Pi_i - \frac{\partial R^*}{\partial \eta_i},
\]

in which

\[
\dot{\eta}_1 = \omega = \dot{z}, \quad \dot{\eta}_2 = \nu = \dot{x}\cos\theta + \dot{y}\sin\theta + h(\dot{\alpha} - \beta\theta);
\]

\[
\dot{\eta}_3 = \omega = -\dot{x}\sin\theta + \dot{y}\cos\theta + h(\beta' + \alpha\theta);
\]

\[
\dot{\eta}_4 = \omega_i = -\beta' - (\alpha + \gamma)\theta, \quad \dot{\eta}_5 = \omega_2 = \dot{\alpha} - \beta\theta;
\]

\[
\dot{\eta}_6 = \omega_{i_1} = \theta, \quad \dot{\eta}_7 = \dot{\psi}, \quad \dot{\eta}_9 = \dot{\Omega}_i = \dot{\gamma}_i \quad (i = 1A).
\]

Here, $h$ is the distance from the center of mass of the body to the $(x_2, y_2)$ plane, and $\gamma$ is the angle between the major axis of the central ellipsoid of the body's inertia and the $x_2$ axis.

Let $A$, $B$, and $C$ be principal moments of inertia of the body, which has a mass $m_1$, $m_k$ is the mass of a wheel, and $I_k$ and $A_k$ are the axial and diametrical moments of inertia of the wheel, respectively. Considering all wheels to be the same, we can then compile expressions for the kinematic energy $T^*$ of the vehicle

\[
T^* = \frac{1}{2} m_1 \sum_{j=1}^{3} \dot{\eta}_j^2 + \frac{1}{2} (A\dot{\eta}_4^2 + B\dot{\eta}_5^2 + C\dot{\eta}_6^2) + \frac{1}{2} \sum_{i=1}^{4} I_i \dot{\eta}_{i+1}^2 + \nonumber
\]

\[
+ m_k \sum_{i=0}^{1} [(g_1 + (-1)^i a_{i+1} \dot{\eta}_j)^2 + (g_2 + (-1)^i b \dot{\eta}_j)^2] + 2A_2 \dot{\eta}_6^2,
\]

the potential energy $U^*$

\[
U^* = m_1 gz + \frac{1}{2} \sum_{i=0}^{1} \sum_{j=0}^{8} C_{ij} \dot{\eta}_j^2 = g_4 + (-1)^i b \beta)
\]

and the energy-dissipation function $R^*$

\[
R^* = \frac{1}{2} \sum_{i=0}^{1} \sum_{k=0}^{8} h_i \dot{\eta}_{k+1}^2.
\]

Here $g_1 = \dot{\eta}_3 + h(\dot{\eta}_4 + \dot{\eta}_5)$, $g_2 = \dot{\eta}_2 + h\dot{\eta}_5$, $g_3 = z - a_\alpha - \ell_1$, $g_4 = z + a_\alpha - \ell_2$, $\ell_1 = \ell_4 + r_0$, $\ell_i$ is the length of the free spring of the shock absorbers, $C_i$ is the stiffness of the front ($i = 1$) and rear ($i = 2$) shock-absorber springs, and $h_i$ is the coefficient of viscous friction in the shock absorbers. The coefficients $\gamma_{ijk}$, which can be determined from ad-