We consider the construction of the positive cone [7] by the method of averaging, which makes it possible to determine oscillatory modes in a many-frequency system with a polynomial nonlinearity and to construct curves dividing identical behavior of the trajectories. The starting point is the set of results given in [1-6].

1. Transformation of the Equations of Motion. We consider an autonomous system describing a certain oscillation process

\[ \dot{x} = A x + \mu X(x), \quad (\cdot) = \frac{d}{dt}. \]  

where \( x(t) \in \mathbb{R}^{2n} \) for all \( t \in \mathbb{R} \), \( A \) is a \( 2n \times 2n \) constant matrix, \( 1 > \mu > 0 \) is a small parameter, \( X(x): \mathbb{R}^{2n} \rightarrow \mathbb{R}^{2n} \) is a vector polynomial of integer powers first order and higher. Suppose that the eigenvalues of the matrix of the linear system corresponding to (1.1) are complex conjugates

\[ \lambda_j, \bar{\lambda}_j = \text{Re} \lambda_j \pm i \text{Im} \lambda_j, \quad (j = 1, \ldots, n). \]  

The conditions for which the variables of (1.1) can be separated into fast and slow variables are known (see [1], Sec. 163): \( | \text{Re} \lambda_k | < < | \text{Im} \lambda_k | \), \( (j, k = 1, \ldots, n) \). We write (1.1) in the form

\[ x = A_1 x + \mu [ A_2 x + \Xi(x) ]. \]  

where \( A_1 \) and \( A_2 \) are \( 2n \times 2n \) constant matrices and \( \Xi(x) \) is a vector polynomial containing terms second order and higher. The unperturbed form of (1.3) has the matrix \( A = A_1 \). Suppose that the roots of the matrix \( A_1 \) are imaginary and \( X(x) = A_2 + \Xi(x) \).

With the help of the nondegenerate linear transformation for (1.3)

\[ y_j = \sum_{k=1}^{2n} a_{jk} x_k, \quad \bar{y}_j = \sum_{k=1}^{2n} \beta_{jk} x_k, \quad x_k = \sum_{(j)} a_{kj} y_j + \bar{a}_{kj} \bar{y}_j, \]  

where \( a_{jk}, \beta_{jk}, \bar{a}_{kj}, \bar{\beta}_{jk} \) are constants, (1.1) is brought to diagonal form

\[ \dot{y}_j = y_j - \mu \bar{y}_j, \quad \bar{\dot{y}}_j = \bar{\lambda}_j y_j + \mu \bar{y}_j, \quad (j = 1, \ldots, n) \]  

where \( Y_j = \sum_{(k)} \beta_{ik} X_k(y, \bar{y}), \quad Y_j = \sum_{(k)} \bar{\beta}_{ik} X_k(y, \bar{y}); X_k(y, \bar{y}) \) is a vector function in which the vector \( x \) is expressed in terms of \( y \) and \( \bar{y} \).
In (1.4) we transform to the variables $\rho$, $\theta$ following [8], p. 96.

\begin{align}
\rho_j &= \gamma_j e^{-i\theta_j}, \quad \gamma_j e^{i\theta_j}, \quad (j = \overline{1,n})
\end{align}

In terms of $\rho$, $\theta$ the equations of motion transform to

\begin{align}
\dot{\rho} &= \mu m(\theta) X(\rho, \theta), \quad \rho \dot{\theta} = \text{Im} \Lambda \rho + \mu k(\theta) X(\rho, \theta),
\end{align}

where $\text{Im} \Lambda$ is an $n \times n$ diagonal matrix and $m(\theta)$ and $k(\theta)$ are $n \times 2n$ matrices with elements

\begin{align}
m_{j0} &= \text{Re} \beta_j \cos \theta_j + \text{Im} \beta_j \sin \theta_j, \quad k_{j0} = \text{Im} \gamma_j \cos \theta_j - \text{Re} \gamma_j \sin \theta_j.
\end{align}

$X(\rho, \theta)$ is a vector function in which $X$ is expressed through $\rho$, $\theta$ as

\begin{align}
x_k &= 2 \sum_{\nu=1}^{n} \rho_{\nu} \left( \text{Re} \alpha_{\nu k} \cos \theta_{\nu} - \text{Im} \alpha_{\nu k} \sin \theta_{\nu} \right) \quad (k = \overline{1,2n}).
\end{align}

Note that in (1.6) the variables $\rho$ are identified with the vector of slow variables and $\theta$ with the vector of fast variables. The components of $\rho$ can be expressed in terms of $x$ and $\theta$ as

\begin{align}
\rho_j &= \left( \sum_{k=1}^{2n} \text{Re} \beta_{jk} x_k \right) \cos \theta_j + \left( \sum_{k=1}^{2n} \text{Im} \beta_{jk} x_k \right) \sin \theta_j \quad (j = \overline{1,n}).
\end{align}

We represent (1.6) in the form

\begin{align}
\dot{\rho} &= \mu R(\rho, \theta), \quad \dot{\theta} = \text{Im} \Lambda \rho + \mu \Theta(\rho, \theta)
\end{align}

and assume the initial conditions

\begin{align}
\rho(0) = \langle \rho(0) \rangle = \rho_0, \quad \Theta(0) = \langle \Theta(0) \rangle = \theta_0.
\end{align}

Here

\begin{align}
R(\rho, \theta) &= \left( R_1(\rho, \theta), \ldots, R_n(\rho, \theta) \right), \quad \Theta(\rho, \theta) = \left( \Theta_1(\rho, \theta), \ldots, \Theta_n(\rho, \theta) \right),
\end{align}

\begin{align}
R_j(\rho, \theta) &= \sum_{k=1}^{2n} m_{jk}(\theta) X_k(\rho, \theta), \quad \Theta_j(\rho, \theta) = \sum_{k=1}^{2n} k_{jk}(\theta) X_k(\rho, \theta) / \rho_j \quad (j = \overline{1,n}).
\end{align}