System of Equations. The problem of capillary equilibrium in porous media, formulated in our previous article [1], is reduced to an infinite system of interlinked differential equations for the values of \( y_i \),

\[
\frac{dy_{2k+1}}{dx} + \left[ \frac{\lambda(1 - S_{2k}) - \frac{1}{3} v(1 - R_{2k})}{1 - Y_{2k}} \right] y_{2k+1} + \frac{1}{3} v \frac{(1 - T_{2k})}{(1 - Y_{2k})^2} y_{2k+1}^2 = \frac{2}{3} v \frac{(1 - R_{2k-1})(1 - Y_{2k})y_{2k}}{(1 - Y_{2k-1})^2}
\]

\[
\frac{dy_{2k}}{dx} + \left[ \frac{\lambda(1 - W_{2k-1}) - \frac{1}{3} v(1 - T_{2k-1})}{1 - Y_{2k-1}} \right] y_{2k} - \frac{1}{3} v \frac{(1 - R_{2k-1})y_{2k}}{(1 - Y_{2k-1})^2} = - \frac{2}{3} v \frac{(1 - T_{2k-2})(1 - Y_{2k-1})y_{2k-1}}{(1 - Y_{2k-2})^2}
\]

with boundary conditions

\[
y_{1}^{(0)} = \delta_{1,1}
\]

and auxiliary functions

\[
\begin{align*}
 r_{2k+1} &= \frac{1 - R_{2k}}{1 - Y_{2k}} y_{2k+1}; & r_{2k} &= \frac{2}{1 - Y_{2k-1}} y_{2k} - \left( \frac{y_{2k}}{1 - Y_{2k-1}} \right)^2 (1 - R_{2k-1}) \\
 t_{2k+1} &= \frac{2}{1 - Y_{2k}} \left( \frac{y_{2k+1}}{1 - Y_{2k}} \right)^2 (1 - T_{2k}); & t_{2k} &= \frac{1 - T_{2k-1}}{1 - Y_{2k-1}} y_{2k} \\
 s_{2k+1} &= \frac{1 - S_{2k}}{1 - Y_{2k}} y_{2k+1}; & s_{2k} &= 0 \\
w_{2k+1} &= 0; & w_{2k} &= \frac{1 - W_{2k-1}}{1 - Y_{2k-1}} y_{2k}.
\end{align*}
\]

The functions denoted by capital letters are sums of the corresponding values

\[
Y_k = \sum_{i=1}^{k} y_i; \quad R_k = \sum_{i=1}^{k} r_i; \quad T_k = \sum_{i=1}^{k} t_i; \quad S_k = \sum_{i=1}^{k} s_i; \quad W_k = \sum_{i=1}^{k} w_i.
\]

System (1) consists of two types of equations; the first pertain to odd cycles, the second to even cycles. Let us clarify the meaning of the individual members of these equations. Let us consider an odd cycle. A contribution to the derivative that describes the change in the probability of filling of the pores is given by the diagrams (see Figs. 1 and 2 [1]) containing junctions of the H, F, and G types. Type Q junctions are not filled in an odd cycle,
therefore the function $W$ does not figure in the equation. When type H junctions are filled, the probability of filling of the pores in the following cross section is reduced; hence the contribution to the derivative from these diagrams is negative. In the equation, they are represented by the factor $\frac{\lambda(1 - S_{2k}) y_{2k+1}}{1 - y_{2k}}$. The factor $\lambda$ describes the probability of sealing, i.e., the improbability of appearance of a H junction; $(1 - S_{2k})$ gives the fraction of junctions which were not filled heretofore, and, consequently, may become filled. The probability of filling of the junction H is linear with respect to $y_{2k+1}$; hence $y_{2k+1}$ appears in the equation to the first power. $(1 - y_{2k})$ is the relative fraction of those pores which remain liquid at a given moment; only they will be discussed. This factor is normalizing in character. In other words, the expression $\frac{y_{2k+1}}{1 - y_{2k}}$ should be considered as the renormalized probability of filling of the pores, i.e., the probability related only to the unfilled pores. The factor $\frac{1}{3} v \frac{(1 - R_{2k}) y_{2k+1}}{1 - y_{2k}}$ describes the contribution from diagrams containing an F junction, moreover, a junction that is filled precisely in the cycle $2k + 1$ under consideration. It is clear that the contribution from such diagrams is positive. The value of the factors that enter into this case is similar to the preceding.

If a G junction is filled along one pore, then the probability of filling of pores in the cross section $x + dx$ is not changed by this in comparison with the cross section $x$. Consequently, such diagrams make no contribution to the equation. Actually, the equation contains no factors that might contain the function $T$ and be linear with respect to $y_{2k+1}$. When a G junction is filled along two pores, the probability of filling in the cross section $x + dx$ decreases. The probability of filling of the junction G along two independent pores is proportional to the second power of the renormalized probability $\frac{y_{2k+1}}{1 - y_{2k}}$. If we also consider the factor $(1 - T_{2k})$, the relative fraction of unfilled junctions, then we obtain

$$\frac{(1 - T_{2k})}{1 - y_{2k}} \frac{1}{3} v y_{2k+1}.$$

The very last factor describes the contribution from diagrams containing the junction F, which in the preceding even cycle $2k$ was filled along one pore $D'$. The second pore $D'$ must remain gas-filled in the cycle $2k + 1$. The factor $(1 - R_{2k-1})$ represents the fraction of F junctions that are not filled by the beginning of the cycle; $2k \frac{y_{2k}}{1 - y_{2k-1}}$ is the renormalized surface of filling in the cycle $2k$, while $\frac{1 - y_{2k}}{1 - y_{2k-1}}$ is the renormalized probability of nonfilling in the cycle $2k$.

The meaning of the factors contained in the equation of even cycles may be clarified quite analogously.

Asymptotic Solution. System (1) was constructed in such a way that the equations of the higher cycles depend on the lower, while the lower do not depend on the higher. The same is also correct for the function of filling. Hence, the system can be gradually "unravelled," beginning with the first equation. Thus we can calculate $y_1(x)$, $y_5(x)$, $y_3(x)$, etc. By breaking off this process at some stage and summing the $y_1(x)$ obtained, we find an approximate value of the function sought, $Y_\infty(x)$. Moreover, the approximate value is closer to the true value, the smaller the distances $x$ considered. It is clear that at sufficiently small $x$, the most significant contribution to $Y_\infty(x)$ will be made by $y_1(x)$ equal to [2].

$$y_1(x) = \frac{\lambda - \lambda x / v}{\lambda e^{\mu x} - \lambda x / v},$$

where $\mu = \lambda - 1/\lambda v$.

When $\lambda = 1/\lambda v$, the solution of the first cycle takes the form

$$y_1(x) = \frac{1}{1 + \lambda x}.$$

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