We consider the parametric excitation of long-wave magnetohydrodynamic oscillations of the m = 1 type in a cylindrical plasma conductor with an alternating, longitudinal, high-frequency current. The plasma cylinder is placed in a constant longitudinal magnetic field and is enclosed in a conducting case. The problem is solved on the basis of the flexible filament model under the assumption of ideal conductivity of the plasma and the case. Hill's method is used to study the stability of the equation with periodic coefficients that describe the oscillations of the filament. Results of computer calculations of the stability increments of oscillations in the first four resonance zones for various values of the parameters of the system are given.

As was shown in [1], when a high-frequency alternating current is used for the dynamic stabilization of a plasma cylinder in a longitudinal magnetic field, parametric excitation of magnetohydrodynamic proper oscillations of the cylinder, that are characterized by the azimuthal wave number m = 1, can occur. In [2] the boundaries of the first two zones of parametric excitation of the m = 1 mode were determined, and an analytic expression for the maximum increment of the buildup of short-wave oscillations was obtained (ka >> 1, where k is the wave number of the perturbation and a is the radius of the cylinder). It is of interest to make a more detailed investigation of the instability in question in the range of long-wave perturbations (ka ≈ 1), which must be excited under experimental conditions (see, for example, [3, 4]). This problem is solved in the present paper. Here we give a numerical calculation of the instability increments of long-wave oscillations of the m = 1 type, excited in a cylindrical plasma conductor by a high-frequency longitudinal current. In contrast to [1, 2], the effect of the conducting case, surrounding the plasma cylinder, is taken into account. The problem is solved on the basis of the flexible filament model under the assumption of ideal conductivity of the plasma and the case. Hill's method [5] is used to investigate the stability of the equation with periodic coefficients that describes the oscillations of the filament. Various possible regimes of the operation of the system are considered.

1. Formulation of the Problem

Suppose that a high-frequency longitudinal current I = I₀ cos ωt flows along the surface of a cylindrical plasma conductor. The conductor is placed in a constant longitudinal magnetic field, equal to B₀ outside and to B₁ inside the plasma, and is enclosed in a conducting case of radius b. The plasma pressure p counterbalances the time-averaged pressure of the magnetic field

\[ \frac{8}{3} p = \frac{B_a^2}{\rho} - \frac{B_t^2}{\rho} + \langle B_z^2 \rangle \]  

Here \( B_a = B_{a0} \cos \omega t = 2I/ca \) is the azimuthal field of the current I on the surface of the filament and the angular brackets denote a time average.
It is convenient to reduce the investigation of the stability of the system under consideration with respect to magnetohydrodynamic perturbations of the \( m = 1 \) type, for which the surface of the conductor is described by the equation

\[
    r = a + \frac{\xi}{r_1(t)} \exp(i k z + i \phi) \quad (\xi \ll a),
\]

to the flexible filament model. The force \( F \) per unit length, perpendicular to the \( z \) axis, acting on the perturbed conductor, is calculated in the magnetostatic approximation, and then we consider the equation for the transverse motion of a length element of the conductor with mass \( M \) per unit length

\[
    M \frac{d^2 \xi}{dt^2} = F
\]

in which the time dependence of the magnetic field, entering into \( F \), is already explicitly taken into account. A comparison with results of a rigorous magnetohydrodynamic analysis (valid for the case of static fields) shows that this model describes the system under consideration sufficiently well if the plasma is considered to be incompressible and the perturbations are long-wave ones.

In the case under consideration the force \( F \) can be found from formulas of [6] and has the form

\[
    F = -\frac{1}{\kappa} \left[ a_0^2 \left( k a B_z \pm B_a \right)^2 + a_1 (k a)^2 B_z^2 - B_a^2 \right] \frac{\xi}{r_1}
\]

Here

\[
    a_0 = \frac{\sigma_0 + \sigma_0 \delta}{1 - \delta}, \quad a_1 = -\frac{K_1(k a)}{k a K_1'(k a)},
\]

\[
    \delta = \frac{K_1'(k a)}{K_1(k a) K_1'(k b)}
\]

\( K_1(x), I_1(x) \) are modified Bessel functions, and a dash denotes differentiation with respect to the argument. When the case is absent, \( \delta = 0 \) and (1.4) reduces to the corresponding expression in [2].

Because of the sinusoidal time dependence of the current \( I \) assumed above, Eq. (1.3) is an equation with periodic coefficients. Upon the substitution \( \omega t = 2 \pi \) it reduces to the standard form of Hill's equation [5] with three terms:

\[
    \frac{d^2 \xi}{dt^2} + \left( \theta_0 + 2 \theta_1 \cos 2 \tau + 2 \theta_2 \cos 4 \tau \right) \xi = 0
\]

Here

\[
    \theta_0 = 4 \left( \omega_s / \omega \right)^2 \left[ (k a)^2 (a_0^2 + \omega_s^2) + 1/4 (a_0^2 - 1) \omega_s^2 \right]
\]

\[
    \theta_1 = \pm 4 \left( \omega_s / \omega \right)^2 k a \omega_s^2 h_s, \quad \theta_2 = 2 \left( \omega_s / \omega \right)^2 (a_0^2 - 1) \omega_s^2
\]

\[
    \omega_s = \nu_s / a, \quad \nu_s = B_s / \sqrt{4 \pi \rho} \quad \text{(where \( \rho \) is the plasma density),}
\]

\[
    h_s = B_s / B, \quad h_a = B_a / B
\]

We note that, in view of (1.1), the velocity \( v_s \) under typical experimental conditions when \( \langle B_a^2 \rangle \ll B_e \) is close to the velocity of magnetic sound in the plasma.

In the region \( k a \ll a / \omega h_s \), where the influence of the case is considerable, the coefficients of Eq. (1.5) differ significantly from the coefficients of the analogous equation of [2]. In particular, \( \theta_2 \gg \theta_1 \), so that the term with \( \theta_2 \) in (1.5) cannot be neglected in our investigation of stability, as was done in [2].

We write the general solution of Eq. (1.5) in the form [5]

\[
    \xi(t) = C_0 e^{i \Omega_0 t} + C_2 e^{i \Omega_2 t}
\]