EQUATION FOR THERMIONIC EMISSION IN A PLASMA

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The equation for thermionic emission from hot cathodes in the presence of a low-temperature plasma is discussed. The distribution function is obtained for the magnitude of the electric field intensity at the surface of the cathode, taking account of the effect of the individual fields of the various ions moving in the near-cathode layer. The thermionic flux density due to fluctuations of the field is found to be significantly higher than that calculated by Richardson's formula with Schottky's correction.

Richardson's classical formula with Schottky's correction for the thermionic current density

$$j_0 = A T^2 \exp \left[ - \frac{e \phi_0}{kT} + \frac{e V E}{kT} \right]$$

where $A$ is the thermoemission constant, $T$ is the cathode temperature, $e$ is the charge of the electron, $\phi_0$ is the work function, $k$ is Boltzmann's constant, and $E$ is the field strength at the cathode, is applicable if the magnitude of the electric field strength is known near the surface of the cathode. In the case of electron emission in vacuo, the field $E$ is defined as the field at a given point of the cathode acting at a given instant. In the case of electron emission in a plasma, in order to calculate the current density formula (1) is also used frequently, in which the quantity $E$ denotes the average electric field at the surface of the cathode, found by solving Poisson's equation on McCohn's assumptions. The use of such a procedure for finding the emission current density in the presence of a plasma is very doubtful. When calculating the thermionic current by formula (1), it was proposed in [1] to take into account the individual fields of the various ions moving in the cathode region, in addition to the central field.

Between the "hot" cathode and the neutral plasma there is a region of an uncompensated space charge of ions. Ions moving toward the cathode create a fluctuating electric field on its surface, the magnitude of which at each point of the cathode depends on the number of ions near it and their disposition. We emphasize that any effect on the magnitude of the electric field at an arbitrary point of the cathode comes only from those ions whose charges are not compensated for this point, i.e., ions located in the cathode screening layer. In the case of a low-temperature plasma, when the concentration of charged particles in the range from $10^{13}$ to $10^{18}$ cm$^{-3}$, only a small number of ions will determine the size and direction of the electric field at an arbitrary point of the cathode; then, the fluctuations of the electric field at the surface of the cathode will be considerable and the use of a value for the central field in formula (1) is very problematical, especially as the dependence of $j$ on $E$ is strongly nonlinear.

In order to calculate the thermionic current density using formula (1), strictly speaking, for an arbitrary point of the cathode it is necessary to know the relation between the electric field strength and the time, $E(t)$. If, however, $E$ is a random quantity which at every finite instant can assume values from its minimum to its maximum $E_*$, then, it is possible to manage without knowing the function $E(t)$. Such a probable approach, obviously, can be taken as acceptable as the field at an arbitrary point of the cathode is created by ions whose coordinates are random values. Thus, in order to describe the thermionic emission...
process in the plasma, it is necessary to know the distribution function of the electric field near the surface of the cathode $f(E)$.

We shall find the distribution function $f(E)$ with the following assumptions:

1) The point of the cathode to be considered is remote from the lateral boundaries of the discharge, i.e., there are no edge effects.

2) The surface of the cathode is plane and without roughness.

3) All ions recombine at the surface of the cathode.

4) The probability of finding an ion in any fixed region of the cathode space is proportional to the volume of this region, and is independent of whether or not the ions are found here.

5) The ion concentration in the cathode region is assumed to be constant.

6) The electric field strength at a given point of the cathode depends only on the location of the ion "nearest" to this point.

We locate the origin of the coordinates at the point of the cathode being considered; the axes $x$ and $y$ are located in the plane of the cathode, and the axis $z$ is directed perpendicular to it on the side of the plasma.

If, at the point with the coordinates $r(x, y, z)$, there is a positive ion with charge $q$, then, at the point $(0, 0, 0)$, taking into account its mirror image, the ion will create a field

$$E = qr q_{rl} r \frac{r - r_{rl}}{r_{rl}^2}$$

The absolute value of the field component along the axis $z$ is equal to

$$E_z = \frac{2q}{r^2} \frac{z}{r}$$

Let us consider two surfaces $S_1$ and $S_2$ for which the equations have the form

$$E = 2q \left( x^2 + y^2 + z^2 \right)^{\nu_1}$$

$$E - dE = 2q \left( x^2 + y^2 + z^2 \right)^{\nu_2}$$

Equations (2) and (3) define a surface with constant components of the field along the axis $z$. The volume of a body enclosed inside the surface $S_1$ is equal to

$$V_1 = \frac{4\pi}{15} \left( \frac{2q}{E} \right)^{\nu_1}$$

The volume of the body enclosed between the surfaces $S_1$ and $S_2$ is equal to

$$V_2 = \frac{2\pi}{5} \frac{(2q)^{\nu_1}}{E^{\nu_2}} dE$$

In order that the component of the electric field strength along the axis $z$ be in the range from $E$ to $E - dE$, it is necessary to satisfy two conditions: there is no ion within the surface $S_1$, and there is one ion between the surfaces $S_1$ and $S_2$, i.e.,

$$p \left( E - dE \leq E_z \leq E \right) = f(E) dE = p_1 p_2$$

where $p_1$ is the probability that there is no ion within the surface $S_1$, and $p_2$ is the probability that there is one ion between the surfaces $S_1$ and $S_2$.

We consider a cube, for which the center of one side coincides with the origin of the coordinates. Suppose that the characteristic dimensions of this cube are such that its volume $V_0$ is considerably greater than the volume $V_1$, i.e.,

$$V_1 / V_0 \ll 1$$

The ratio $V_1/V_0$ becomes equal to 0.1 when $E \sim 10^4$ V/cm, i.e., with those fields when it no longer exerts any effect on thermoemission, so long as the dimensions of the cube do not exceed a value of the