NUMERICAL AND ASYMPTOTIC SOLUTION
OF THE PROBLEM CONCERNING THE COMPLETE
STABILIZATION OF A BOUNDARY LAYER

S. A. Gaponov and A. A. Maslov

The numerical method given in [1] is used here for calculating the temperatures of complete stabilization for a supersonic boundary layer at a flat plate with the boundary condition \( \theta(0) = 0 \), where \( \theta \) denotes the amplitude of temperature perturbations. According to the results, the conclusion in [2] that there exist two regions of complete stabilization is wrong. The asymptotic method used in [2] is analyzed here. It is shown that two regions of complete stabilization appear to exist, because the equations used in [2] had been set up for the viscous case and, therefore, are not applicable at low surface temperatures. The results of this analysis are confirmed by direct numerical integration.

1. A numerical method has been proposed in [1] for solving problems concerning the complete stabilization of a supersonic boundary layer subject to small two-dimensional perturbations.

The calculations were performed only for the boundary condition

\[
\theta'(0) = 0
\]

with regard to temperature, where \( \theta \) denotes the amplitude of temperature perturbations and the prime indicates a derivative with respect to the coordinate normal to the surface.

Of practical interest is the problem concerning the complete stabilization of a supersonic boundary layer also under the condition that

\[
\theta(0) = 0
\]

Such a boundary condition was stipulated in [2] for calculating the temperatures of complete stabilization. The results in [2] were obtained by the asymptotic method, and they may be unreliable at least within the \( M \sim 2 \) range of Mach numbers (see [1]). It is necessary to refine the results of [2], therefore, this will be done here by the calculation method given in [1]. The viscosity coefficient is assumed proportional to the temperature (\( \mu = T \)), the Prandtl number \( \sigma = 0.75 \), and the adiabatic constant \( \gamma = 1.4 \). The results of calculations for various values of the Mach number \( M \) are shown in Fig. 1: surface temperatures \( T_w \) at which complete stabilization occurs along the first neutral curve (1), and along the second neutral curve (2) (the existence of two neutral curves corresponding to intensive cooling of the surface has been demonstrated in [1]). For comparison, the results obtained in [2] by the asymptotic method are shown here with a dashed line. The comparison indicates a wide discrepancy between the results based on the numerical method and those based on the asymptotic method.

The numerical method has yielded one region of complete stabilization, which is bounded by curve 1 for \( M < 3.2 \) and by curve 2 for \( M > 3.2 \). The asymptotic method [2] has yielded two such regions. The first one is bounded by a curve (dashed line) consisting of two branches which have been labeled I and II in Fig. 1. Branch I coincides with curve 1 for \( M \leq 1.4 \). The second region is bounded by curve III. In order to...
understand the causes of such a discrepancy, we will thoroughly analyze the feasibility of solving the problem of complete stabilization by the asymptotic method.

2. The following system of equations has been derived in [3] for the perturbation amplitudes:

\[
\frac{i(U - c) T}{T'} \psi = - \frac{iP}{\gamma M^2} + \frac{\mu}{3R} \psi'' \tag{2.1}
\]

\[
\frac{i(U - c) a \psi}{\gamma M^2} = - \frac{P'}{\gamma M^2} + \frac{\mu}{3R} c^2 \psi'' \tag{2.2}
\]

\[
i(U - c) r T - \frac{T'}{T} \psi + i \psi' = 0 \tag{2.3}
\]

\[
i(U - c) 0 + \frac{T'}{T} \psi = - (\gamma - 1) (i f + \psi') + \frac{\gamma \mu}{3R} \theta' \tag{2.4}
\]

\[
P = r T + i \theta \tag{2.5}
\]

Here, \( U \) and \( T \) are the time-average velocity and temperature, respectively, \( f, \alpha \psi, r, \) and \( P \) are perturbations of the longitudinal and the transverse velocity, of the temperature, of the density, and of the pressure; \( \gamma \) is the adiabatic constant, \( M \) is the Mach number, \( \mu \) is the velocity coefficient, \( R \) is the Reynolds number, \( \alpha \) is the wave number of a perturbation, and \( c = c_r + ic_\i \) is the phase velocity of a perturbation. In deriving system (2.1)-(2.5) it has been assumed that a perturbation is an exponential function of the longitudinal coordinate \( x \) and of time \( t \): \( \exp i \alpha (x-ct) \).

Let the solution of this system satisfy the following three boundary conditions at the surface:

\[
f_\psi = \psi_w = \theta_w = 0 \tag{2.6}
\]

If the product \( \alpha R \) is sufficiently large, then, the fundamental system of solutions can be made up of the solution to the inviscid equations \( \{ \Phi, F, \psi \} \) and two linearly independent viscous solutions \( \{ \varphi_3, f_3, \theta_3 \} \) and \( \{ \varphi_5, f_5, \theta_5 \} \) [4]. Condition (2.6) will then yield the following relation at the surface [4]:

\[
\frac{\Phi_w}{\varphi_w} = \left[ \frac{\varphi_{3w}}{\varphi_{bw}} + \frac{\varphi_{5w}}{\varphi_{bw}} \left( \frac{\gamma - 1}{2} \right) \frac{\theta_{3w}}{\theta_{bw}} \right] - (\gamma - 1) M^2 \left( \frac{\varphi_{3w}}{\varphi_{bw}} + \frac{\varphi_{5w}}{\varphi_{bw}} \right) \left( \frac{1}{1 - i/c^2} \right)
\]

\[
- i (\gamma - 1) \frac{1}{2} \left( \frac{U_w'}{r} - \frac{T_w'}{T_w} \right) \left( \frac{\varphi_{3w}}{\varphi_{bw}} + \frac{\varphi_{5w}}{\varphi_{bw}} \right) \left( \theta_{3w} - \theta_{bw} \right)
\]

The left-hand side of Eq. (2.7) is usually written as [5]

\[
\frac{\Phi_w}{\varphi_w} = \frac{\Phi_w}{\varphi_w} = \frac{\psi_{3w}}{\psi_{bw}} \frac{\psi_{5w}}{\psi_{bw}} \frac{1}{1 - i/c^2} \tag{2.8}
\]

During complete stabilization, \( \alpha = 0 \) and \( c = 1-M^{-1}, \) and in order to determine the critical surface temperature in (2.8), one needs only the value of \( v(T_w) \) [5] which satisfies the equality

\[
v(T_w) = - \frac{\pi}{\eta} \frac{U_w'}{T_w} \left( \frac{T_w'}{T_w} \right) \left( \frac{U_w'}{T_w} \right) \left( \frac{T_w'}{T_w} \right) \left( \frac{U_w'}{T_w} \right)
\]

within sufficient accuracy.