The spectrum of the characteristic surface oscillations and stability of a plasma in a strong high-frequency (hf) electric field are studied. It is shown that inhomogeneity of the plasma leads to spatial dispersion and to specific damping of stable oscillations of a plasma in an external hf field, the frequency of which greatly exceeds the plasma frequency. A systematic theory of parametric resonance at the frequency of the electronic surface oscillations is developed taking account of the inhomogeneity of the plasma.

1. The dispersion theory of surface waves of a plasma in a strong high-frequency (hf) field [1] has been developed for the case of a homogeneous plasma with a sharp boundary. In actual experiments, the approximation of a homogeneous plasma with a sharp boundary is not always justified. It is known [2, 3] that inhomogeneity of the plasma has a significant influence on the spectrum of hf surface oscillations. Thus, in the case where the characteristic dimension of an inhomogeneity near the plasma boundary greatly exceeds the Debye radius, the spatial dispersion and damping coefficient for hf surface waves is completely defined by plasma inhomogeneity effects. From the theory of parametric resonance in an unbounded homogeneous plasma [4], it is known that for strong hf fields the occurrence of spatial dispersion of plasma waves substantially changes the picture of plasma instabilities. Below, it is shown that analogous effects also occur for parametric resonance at the frequency of hf surface waves in a bounded inhomogeneous plasma. It is established that an aperiodic instability arises not only at a frequency of the external field \( \omega_0 \), lower than the frequency of the surface waves \( \omega_p/(1 + \epsilon_0)^{1/2} \), but also for \( \omega > \omega_p/(1 + \epsilon_0)^{1/2} \). In addition, situations are possible in which the parametric instability is dissipative.

Besides studying the peculiarities of the parametric resonance, the present work also investigates the influence of plasma inhomogeneities on the spectrum of unstable surface oscillations in an external hf field.

2. We consider a plasma with density \( n(z) \) rapidly rising in a transition layer \( 0 \leq z \leq a \) and changing relatively slowly for \( z > a \), so that the characteristic dimension of the inhomogeneity

\[
L = (\theta \ln n(z) / \partial z|_{z=a})^{-1} \gg a.
\]

For such conditions, in the plasma there exist weakly damped surface waves with wave vector \( k_\parallel \), directed along the plasma boundary and satisfying [2, 3]

\[
a^{-1} \gg k_\parallel \gg L^{-1}.
\]  

(2.1)

We assume that the external electric field vector is oriented along the plasma boundary. For sufficiently large \( k_\parallel \), when the region of field localization of the surface wave (equal to \( 1/k_\parallel \)) is much less than the penetration depth of the external field \( c/(\omega_p^2 - \omega_0^2)^{1/2} \), the latter may be considered homogeneous:

\[
E = E_0 \sin \omega_0 t.
\]

The dispersion relations for the lf (with frequency \( \omega \ll \omega_0 \)) and the hf (with frequency \( \omega \approx n\omega_0 \)) surface waves in a strong hf field have the form.
\[ R^{(n)} = [1 + \varepsilon + \delta \varepsilon^{(n)}(a)] \left\{ 1 - \frac{1}{2k} \frac{\partial}{\partial z} \delta \varepsilon^{(n)}(z) \right\}_o + \]
\[ - k \sum_{n=\infty} \int_{0}^{\infty} \frac{dz}{\delta \varepsilon^{(n)}(z)} - k \sum_{n=\infty} \int_{0}^{\infty} \frac{dz}{1 + \delta \varepsilon^{(n)}(z)} \times \]
\[ \left[ \frac{\delta \varepsilon^{(n)}(z)}{1 + \delta \varepsilon^{(n)}(z)} - \frac{\delta \varepsilon^{(n)}(z)}{1 + \delta \varepsilon^{(n)}(z)} \right] \left[ 1 + \delta \varepsilon^{(n)}(z) \sum_{n=\infty} \frac{J^2}{1 + \delta \varepsilon^{(n)}(z)} \right]^{-1} + \]
\[ + \frac{1}{2k} \frac{\partial}{\partial z} \left[ 1 + \delta \varepsilon^{(n)}(z) \right]_{z=a} + k \varepsilon \int_{0}^{\infty} \frac{dz}{1 + \delta \varepsilon^{(n)}(z)} + \]
\[ + k \int_{0}^{\infty} dz \left[ 1 + \delta \varepsilon^{(n)}(z) \right] + \frac{1 + \delta \varepsilon^{(n)}(z)}{2k} \frac{\partial}{\partial z} \]
\[ \delta \varepsilon^{(n)}(z) = - \omega_{Le}^2(z) / (\omega + n\omega_0)^2, \quad \tau_E = \varepsilon E_0 / n\omega_0^2. \]

Here \( \varepsilon_0 \) is the dielectric permeability of the medium surrounding the plasma: \( \delta \varepsilon^{(n)}(z) \) is the contribution of particles of type \( \alpha \) in the linear dielectric permeability of a cold plasma; \( J_n \) is the Bessel function of argument \( k_{||} r_E \) and \( r_E \) is the amplitude of electron oscillations in the hf field.

3. Using the dispersion relation (2.2), we first examine the spectra of the weakly damped surface oscillations in the case of external fields with frequencies much greater than the plasma frequency, \( \omega_\| = [\omega_{Le}^2(\alpha) + \omega_{Li}^2(\alpha)]^{1/2} \). In this case there may exist both hf and If surface waves of frequency \( \Omega \) and damping coefficient \( \gamma' \) given by:

\[ \Omega^2 = \omega_\|^2 \left\{ 1 - \frac{\omega_\|^2}{1 + \varepsilon} + \frac{k_{||}}{1 + \varepsilon} \int_{0}^{\infty} \frac{dz}{\varepsilon(\omega, z)} \right\} + \frac{1}{2k} \frac{\partial}{\partial z} \left[ \left( \omega_\|^2 - \omega^2(\omega, z) \right) \right]_{z=a} \] (3.1)

\[ \gamma' = \frac{\pi \varepsilon_0 \omega_0^2 k_{||}}{4(1 + \varepsilon)} \int_{0}^{\infty} dz \varepsilon(\omega, z). \] (3.2)

For If oscillations, the frequency of which is significantly less than the plasma frequency, the following must be used for \( \omega_\|^2 \) and \( \varepsilon(\omega, z) \):

\[ \omega_\|^2 = [1 - J_\|^2 (k_{||} r_E)] \frac{\omega_{Le}^2(\alpha)}{1 + \varepsilon}, \]

\[ \varepsilon(\omega, z) = 1 - [1 - J_\|^2 (k_{||} r_E)] \frac{\omega_{Le}^2(\alpha)}{\omega^2}. \] (3.3)

while for hf oscillations they have the form

\[ \omega_\|^2 = \frac{\omega_{Le}^2(\alpha)}{1 + \varepsilon}, \quad \varepsilon(\omega, z) = 1 - \frac{\omega_{Le}^2(\alpha)}{\omega^2}. \] (3.1)

To an accuracy of second order in the mass ratio of electrons to ions, the hf field does not change the spectrum of the hf surface oscillations. Hence, the second and third terms in the curly brackets of Eq. (3.1) and the damping factor [Eq. (3.2)] may be evaluated using the dispersion relation obtained by Stepanov [3], and the last term of Eq. (3.1) corresponds to the correction considered by Romanov [2] for plasma inhomogeneities in the absence of an hf field.

We note that the choice of the point \( a \) bounding the transition layer is based only on condition (2.1). For this, the expression for the frequency is independent of the choice of \( a \) in the region of slow density change \( a \ll L \) since everywhere in this region \( \varepsilon(\omega, z) \approx -\varepsilon_0 \).

4. As the frequency \( \omega_\|^2 \) of the external field is decreased and its harmonic \( n\omega_\|^2 \) approaches \( \omega_\|^2 \) [Eq. (3.4)] a parametric resonance is excited which leads to a growth rate \( \gamma \) for If (with frequency \( \omega \)) and hf (with frequencies \( \omega \pm n\omega_\|^2 \)) surface oscillations. From Eq. (2.2) for this case we obtain

\[ (\omega + i\gamma)^2 - \frac{\omega_{Le}^2(\alpha)}{1 + \varepsilon} \frac{J_\|^2 (k_{||} r_E)}{\Delta^2} - (\omega + i\gamma)^2 = 0. \] (4.1)

Here \( \Delta = n\omega_0 - \Omega \), where \( \Omega \) is defined by Eqs. (3.1) and (3.4).