DIFFUSION PROBLEMS IN THE LINEAR THEORY
OF GASDYNAMIC AND CHEMICAL LASERS

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The system of linear equations of multicomponent convective diffusion is reduced consistently with allowance for the features of the relevant laser devices. A criterion for the realization of a purely diffusion regime is established. A method of diagonalization of the diffusion coefficient matrix is given; it reduces the multicomponent problem to a series of one-component problems. The superposition of relaxation modes is discussed. A diffusion type hydrofluoric laser illustrates the influence of angular asymmetry of the particles on the output power of the radiation.

1. If the kinetic processes that determine the parameters of gasdynamic and chemical lasers (see the reviews [1-3]) are to be established precisely, it is frequently necessary to take into account the diffusion characteristics of a nonequilibrium flux. A considerable number of laser systems have been developed in which the diffusion transport of atoms and molecules is the dominant kinetic process [4, 5].

By using simplified estimates of the diffusion contributions to the relaxation times and also other parameters that directly determine the gain, power, and efficiency of lasers, one can seriously distort the results of calculations [6] or even arrive at a qualitative contradiction to experimental data [7]. In this paper, I consider linear boundary-value diffusion problems whose solutions must be taken into account. These problems admit either a direct analytic or simple numerical solution, so that the results can be used to improve the model theories of lasers like those in [8-10].

2. In the framework of the phenomenological description of vibrational relaxation of the levels, combined in "blocks", of the basic laser components (for example, in the Landau–Teller approximation [8] and the Courant–Friedrichs–Levi stability conditions [11]) one can proceed from the following system of linear equations of convective diffusion:

\[
\frac{\partial n_i}{\partial t} + \mathbf{v} \cdot \nabla n_i - \sum_{k=1}^{p} D_{ik} \nabla^2 n_i + \alpha_i n_i = 0, \quad \mathbf{v} = \sum_{i=1}^{p} n_i \mathbf{v}_i.
\]

(2.1)

Here \( \mathbf{v} \) is the vector of the mean velocity of the flux; \( \alpha_i \) are dissipative coefficients; and \( D_{ik} \) are diffusion coefficients which, to a good approximation, can be regarded as scalar quantities related to one another by the Onsager relations.

We find a criterion that enables one to distinguish a purely diffusion regime of operation of the laser. If \( L_\parallel \) and \( L_\perp \) are the characteristic longitudinal and transverse dimensions of the laser channel (or rather that part of it in which the diffusion process is important), the criterion can be formulated as

\[
t_c \geq 10 t_d, \quad t_c = L_\parallel / v, \quad t_d = L_\perp^2 / \langle D_{ik} \rangle
\]

(2.2)

where \( t_c \) and \( t_d \) are the times of convective and diffusion passage of a particle. Introducing the Schmidt and Reynolds numbers for the flux, \( Sc \) and \( Re \) respectively, we can rewrite (2.2) as

\[
Sc \ Re \leq 0.1 \ L_\parallel / L_\perp
\]

(2.3)
The opposite inequality corresponds to the fast-flow regime [12]. Neither this nor the intermediate case [13] will be considered.

Apart from the neglect of the second term, on the basis of (2.3), a further formal simplification of (2.1) can be achieved by the change of variables

\[ m_i = n_i e^{\kappa_t} \]

Then (2.1) is replaced by

\[ \frac{\partial n_i}{\partial t} = \sum_{k=1}^{p} D_{ik} \nabla^2 m_k \]

(2.5)

\[ D_{ik}^* = D_{ik} e^{(\alpha_k - \alpha_i)t} \]

(2.6)

The literature data (see, for example, [3, 6, 10]) enable one to make a comparative estimate of the quantities \( \alpha_i \) and the differences \( \alpha_1 - \alpha_k \) in the exponential in (2.6). It is found that a large number of practically interesting cases are characterized by

\[ |\alpha_i - \alpha_k| < \xi_a^{-1} \]

(2.7)

i.e., to a fair degree of accuracy one can ignore the difference between the dissipative coefficients in the multicomponent problem and proceed from the system of equations

\[ \frac{\partial n_i}{\partial t} = \sum_{k=1}^{p} D_{ik} \nabla^2 n_k - \alpha_i n_i \]

(2.8)

3. In the thermodynamics of irreversible processes in the solution of problems described by equations of the type (2.8), one attempts, using different approximations, to decouple the system or ignore the nondiagonal terms of the matrix \( D_{ik} \) of diffusion coefficients. As a rule, such procedures are hard to justify and their use introduces uncontrollable errors into the calculations [14, 15].

Note that (2.8) is transformed into a system of \( p \) independent equations by the diagonalization of \( D \). We consider an auxiliary nonsingular square matrix \( g \) with elements \( g_{ik} \) whose rank is the same as that of \( D \). Multiplying the equations of the system (2.8) by the corresponding elements of \( g_{ij} \), summing over \( i \), and also replacing the subscripts \( i \) by \( k \) in terms with single summation, we find

\[ \frac{\partial}{\partial t} \sum_{k=1}^{p} g_{kj} n_k = \text{div} \text{ grad} \left( \sum_{k=1}^{p} g_{kj} D_{ik} n_k - \alpha_i n_i \right), \quad j = 1, 2, ..., p \]

(3.1)

We introduce the notation

\[ G_j = \sum_{k=1}^{p} g_{kj} n_k, \quad H_j = \sum_{i=1}^{p} \frac{1}{g_{ij}} \sum_{k=1}^{p} g_{ij} D_{ik} \]

(3.2)

after which the system (3.1) can be written in the form

\[ \frac{\partial G_j}{\partial t} = H_j \nabla^2 G_j - \alpha_j G_j, \quad j = 1, 2, ..., p \]

(3.3)

where the coupled terms are absent. The elements of the diagonalized matrix, \( H_j \), can be found from the condition

\[ \det (D_{ik} - H_\delta_{ik}) = 0 \]

(3.4)

The solution of the system of equations (2.8) is

\[ n_i = \sum_{j=1}^{p} g_{ij}^{-1} G_j \]

(3.5)

where \( g^{-1} \) is the inverse matrix.

Returning to the more complicated case, when (2.7) does not hold, we note that an investigation was made of the convergence of the iterative scheme in accordance with which coefficients \( \alpha_j \) of the form

\[ \alpha_j = \frac{1}{c_j} \sum_{k=1}^{p} \alpha_k g_{kj} n_k \]

(3.6)

are introduced in (3.3) and in the first approximation the \( n_k \) are found using (3.5). As a rule, more than two iterations are not required.