The analytical solutions obtained in the manner described here make it possible to analyze the entire multiplicity of frontal regimes in the multicomponent \((n \geq 3)\) dynamics of adsorption \((c_{0m} > c_m^0)\) and desorption \((c_{0m} < c_m^0)\) for various values of the concentration \(c_{0m}, c_m^0 (1 \leq m \leq n)\).

LITERATURE CITED


A SET OF STEADY-STATE SOLUTIONS OF THE EVOLUTION EQUATION FOR PERTURBATIONS IN ACTIVE-DISSIPATIVE MEDIA

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Steady-state periodic solutions have been calculated numerically. It is demonstrated that an even set of such solutions comes about as a consequence of a successive cascade of bifurcations.

In recent times, researchers into the wave processes in nonconservative media have expressed great interest in an equation of the form

\[ H_t + 4HH_x + H_{xx} + H_{xxxx} = 0. \]  \hspace{1cm} (1)

This interest is generated by the fact that in terms of form it is one of the simplest nonlinear equations which could possibly be imagined, so that with its appearance in the simulation of the nonlinear behavior of perturbations for a rather large class of active-dissipative media it functions for the latter in as extensive a role as the well-known KdV equation for conservative media.

Thus, in the description of the waves at the surface of a liquid film flowing freely down an inclined plane, such an equation has been derived in [1, 2], for the counterflow motion of a film and a gas we find the derivation of such an equation in [3], and for the perturbations at the boundaries separating two viscous liquids in a horizontal channel, the derivation of the equation is to be found in [4].

Linear stability analysis demonstrated that the trivial solution \(H = 0\) of Eq. (1) is unstable relative to perturbations of the form \(\exp[\imath\omega(x - ct)]\) with wave numbers \(\omega < 1\) (perturbations with \(\omega > 1\) are attenuated). The growth of such perturbations over time can be curtailed through the action of nonlinear effects, as a result of which steady-state nonlinear regimes are formed.
It follows from linear stability theory that the periodic solution of infinitely small amplitude branches from the trivial solution of Eq. (1) when \( \alpha = 1 \). In its vicinity, the solution of small but finite amplitude can be achieved analytically in the form of a series over the small parameter, which is frequently represented by the amplitude itself. It is extended into the region of linear instability \( \alpha < 1 \), i.e., a soft type of branching.

For the steady-state traveling wave \( H(\xi) \) \( (\xi = x - ct) \) Eq. (1) is written in the form

\[
-cH_{\xi} + 4HH_{\xi} + H_{\xi\xi} + H_{\xi\xi\xi\xi} = 0. \tag{2}
\]

In finding the periodic solutions of Eq. (2) with a wavelength \( \lambda = 2\pi/\alpha \), in light of the fact that it is invariant relative to the transformations

\[
H \rightarrow -H, \quad \xi \rightarrow -\xi, \quad c \rightarrow -c, \quad H \rightarrow H + \text{const}, \quad c \rightarrow c + 4 \text{const}, \tag{3}
\]
we will limit ourselves to an examination only of those for which \( c \geq 0 \), \( \frac{1}{\alpha} \int_0^\lambda H d\xi = 0 \). Thus, we arrive at the boundary-value problem in which the eigenvalues are the phase velocity \( c \) of the wave, and where the parameter is the wave number \( \alpha \).

The periodic solutions of Eq. (2) with noticeable amplitude are found numerically. For this, they are presented in the form of a Fourier series

\[
H = \sum_{n=-\infty}^{\infty} H_n \exp[i\alpha n \xi]. \tag{4}
\]

Since \( H \) is a real function, then \( H_n = \overline{H_n} \) (the bar indicates the complex conjugacy operation). Leaving the first \( N \) harmonics in Eq. (4), we will substitute it into (2). Having equated the coefficients for identical exponents to zero, we obtain a system of \( N \) complex equations for the real unknown \( c \) and for \( N \) complex \( H_1, \ldots, H_N \):

\[
(-i\alpha n - \alpha^2 n^2 + \alpha^4 n^4)H_n + 2i\alpha n \sum_{m=-n-N}^{N} H_m H_{n-m} = 0, \quad n = 1, \ldots, N. \tag{5}
\]

In view of the invariance of Eq. (2) relative to the displacement of the coordinate

\[
\xi \rightarrow \xi + \text{const}, \tag{6}
\]
the origin can be set so that, for example,

\[
\text{Re}(H_1) = 0. \tag{7}
\]

With consideration of (7), system (5) is defined. We made use of the Newton method in the numerical solution of this system. In concluding series (4) we chose the number of harmonics so as to fulfill the relationship \( |H_N| / \sup |H_n| \leq 10^{-3} \). For this, the number \( N \) in the calculations had to be changed in dependence on \( \alpha \) in limits from 10 to 40.

The basic difficulty in the solution of Eq. (2) by this method involves the specification of the initial approximation, sufficiently close to the solution. For the first set of solutions, branching from the trivial with \( \alpha = 1 \), we used the analytical solution. Movement along the parameter \( \alpha \) is achieved in continuous fashion, i.e., the wave-number interval is chosen so that in using the initial approximation of the earlier-found solution it will enter the region of convergence.

It has been determined in [5] that the first set of solutions can be continuously extended in the direction of smaller wave numbers to values of \( \alpha_x = 0.4979 \). At this point each odd harmonic of series (4) is equated to zero. As a result, we have a solution with a wave number \( \alpha = 2\alpha_x = 0.9958 \), and here it develops that it coincides with the earlier-derived \( \alpha \). Thus, this set of solutions closes on itself. Let us note that for all \( \alpha \) out of the region of existence of solutions there exists for this family of solutions a phase velocity \( c = 0 \). It follows from (3) that such solutions are antisymmetric. In the present study we will limit ourselves to the examination of those solutions for which \( c = 0 \). With this purpose in mind, we will use a regular procedure to undertake bifurcational analysis of branching from the first set of periodic steady-state solutions of Eq. (1). We demonstrate how an even set of such solutions appears. A portion of the set has been found in [6, 7]. Let us