OPTIMIZATION OF THE PARAMETERS OF A CO$_2$
GASDYNAMIC LASER

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In order to maximize the gain coefficient of a CO$_2$–N$_2$–He gasdynamic laser, a total optimization of the stagnation parameters, the concentration coefficient, and the parameters which determine the shape of the nozzle (assuming a quasi-one-dimensional flow model) was carried out. The dependence of the optimized parameters on the limitations imposed by the stagnation temperature of the flow and the critical cross section have been determined.

Solutions for the population inversion of CO$_2$ molecules and the gain coefficient for a gas mixture consisting of CO$_2$, N$_2$, He, and H$_2$O expanded by a nozzle have been discussed by many authors [1-11] (see also the references in these works).

The basic assumptions have been established for the determination of the rotational–rotational and rotational–translational exchange processes in these mixtures.

The composition and stagnation parameters of the gas mixture, however, have been varied for fixed nozzles. The results of theoretical solutions, obtained previously, agree well with the experiments. The results of these computations and a series of experiments have been successful in pointing out which quantities depend on the inverted population and the gain coefficient, and the degree of their effect. To a much lesser degree, the range of values of the stagnation parameters, the concentration component of the gas mixture, and the nozzle parameters, which are all necessary in order to obtain the results at conditions near optimal, have been determined.

In order to determine the gain coefficient to a much higher accuracy, it is necessary to include the optimization of the stagnation parameters of the gas mixture and the contour of the nozzle which is used.

In this paper, rotational relaxation is examined in the CO$_2$–N$_2$–He mixture, for which the following reaction scheme has been assumed.

The rotational–rotational exchange for the collision of CO$_2$ molecules with some molecule between the $v_1$-, $v_2$-, and $v_3$- modes progresses as follows:

\begin{align}
 hv_3 & \rightleftharpoons hv_1 + hv_2 + \Delta E_1 \\
 hv_3 & \rightleftharpoons 3hv_3 + \Delta E_2 \\
 hv_1 & \rightleftharpoons 2hv_3 + \Delta E_3
\end{align}

For the collision of CO$_2$ molecules with N$_2$ molecules, the exchange of rotational energy proceeds according to

\[ hv_3 \rightleftharpoons h v_4 + \Delta E_4 \]

The exchange of energy between the rotational and translational–rotational motion for the CO$_2$ and N$_2$ molecules struck by some molecules proceeds as

\[ h\nu_i \approx \Delta E_i \]  
\[ h\nu_i \approx \Delta E_i \]  

Here \( \nu_1, \nu_2, \nu_3, \) and \( \nu_4 \) are the characteristic frequencies of the symmetrical valence bond of the doubly-degenerate deformation and the asymmetrical valence bond type of the rotational molecules of CO\(_2\) and N\(_2\), respectively; \( \Delta E_i \) is the value of the energy of the translational-rotational motion for the corresponding processes.

We will make use of the usually accepted and substantiated assumptions in other papers. First we will assume that inside each partially rotational degree of freedom of the CO\(_2\) and N\(_2\) molecules, there exists a local thermodynamic equilibrium (each mode is interpreted as an oscillator with an infinite number of steady-state levels). For this case, each rotational mode can have a corresponding rotational temperature \( T_i (i = 1, \ldots, 4) \). Second, we will assume that \( T_1 = T_2 \) and \( T_1 = 2T_2 \), \( \theta_i = h\nu_i / k \). This is caused by the strong resonant interaction (3) due to the closeness of the energy of the first level of the \( \nu_1 \)-mode and the second level of the \( \nu_1 \)-mode (Fermi resonance). Third, we will assume that the gas is ideal, neglecting such effects as viscosity, thermal conductivity, and diffusion.

As is usually done, we will also assume that the rotational degrees of freedom are excited classically.

The equations for the conservation of mass and impulse energy and the equation for the state of the quasi-one-dimensional flow model assume the form

\[ A\rho u = A\beta u_0 \]  

\[ \rho u \frac{du}{dx} + \frac{dP}{dx} = 0 \]  

\[ \left( \frac{5}{2} + \alpha_1 + \alpha_2 \right) kT + \alpha_1 \sum_{i=1}^{2} h\nu_i \epsilon_i + \alpha_2 h\nu_4 \epsilon_4 + \frac{mnu^2}{2} = \text{const} \]  

\[ p = \rho \frac{k}{m} T \]

This is a closed system for the relaxation equations, which, allowing for the above-mentioned assumptions, has the form

\[ \frac{de_2}{dx} = \frac{2a}{m} (1 + e_2^2) \left( 3e_2^2 + 6e_2 + 2 \right)^{-1} \left[ \sum_{i=1}^{3} a_i K_i (2 \rightarrow 0) (1 - e^{-\frac{\epsilon_i}{T_i}}) (\epsilon_0 - \epsilon_2) - \frac{3}{8} \left( q_{i1} + q_{i2} \right) \right] \]  

\[ \frac{de_1}{dx} = \frac{p}{mu} \left[ \frac{1}{8} \left( q_{11} + q_{12} \right) + \alpha_2 q_{32} \right] \]  

\[ \frac{d\epsilon_1}{dx} = \frac{\rho}{m} \left[ \sum_{i=1}^{3} a_i K_i (3 \rightarrow 2) (1 + e_2) e^{-\Delta E_i / kT} - e_3 (2 + e_3) \right] \]  

\[ q_{11} = \frac{3}{1} \sum_{i=1}^{3} a_i K_i (3 \rightarrow 2) [e_2 (1 + e_2) e^{-\Delta E_i / kT} - e_3 (2 + e_3)] \]  

\[ q_{12} = (1 + e_2)^{-1} \sum_{i=1}^{3} a_i K_i (3 \rightarrow 1, 2) [e_2 (1 + e_2) e^{-\Delta E_i / kT} - e_3 (2 + e_3)] \]  

\[ q_{32} = K (3 \rightarrow 4) [e_3 (1 + e_3) e^{-\Delta E_i / kT} - e_3 (1 + e_3)] \]

Here \( \alpha_j (j=1, 2, 3) \) is the molar concentration of CO\(_2\), N\(_2\), and He, respectively; \( \rho, u, T, P, \) and \( A \) are the density, velocity, temperature, gas pressure, and cross-sectional area of the nozzle, \( m = \sum m_j \alpha_j ; \) \( m_j \) is the mass of the \( j \)-th type of molecule; \( e_2 = 2 (e^{h\nu_2 / kT} - 1)^{-1}; \) \( e_3 = (e^{h\nu_3 / kT} - 1)^{-1}; \) the quantity \( \epsilon_{ij} \) corresponds to the value of \( \epsilon_i \) for \( T_i = T; \) \( K_i (3 \rightarrow 1, 2), \) \( K_i (3 \rightarrow 4), \) \( K_i (2 \rightarrow 0), \) and \( K_i (4 \rightarrow 0) \) are the rates of the respective reactions (1), (2), (4)–(6) for unit molecular concentrations. The values of the reaction rates have been approximated according to [13, 14].

The surface of the diffuser part of the axisymmetrical nozzle is given in the form

\[ A(x) = \begin{cases}  \frac{A_1 + ax^2 + bx^3}{\sqrt{A(l)}} + c(x-l)^2, & 0 \leq x \leq l \\ 0, & x > l \end{cases} \]  

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