FLOW OF A BINARY GAS MIXTURE WITH ARBITRARY
ACCOMMODATION OF THE TANGENTIAL MOMENTUM

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The system of BGK (Bhatnagar, Gross, Krook) equations describing the isothermal flow of a binary gas mixture in a capillary with arbitrary accommodation of the tangential momentum is solved by the Bubnov-Galerkin method. General expressions are given for the kinetic thermodynamic coefficients which are valid in the whole range of Knudsen numbers and have the correct free-molecule and viscous limits. The diffusion-slip coefficients calculated by using test values of the fraction of diffuse reflection, are compared with the experimental results.

A number of phenomena, such as diffusophoresis and mixture separation in a flow, exists in rarefied gasdynamics, which cannot be described by classical hydrodynamics based on the solution of the Boltzmann equation by the Chapman-Enskog and Grad methods even in the viscous mode limit. Only the solution of the Boltzmann equation and its models with the boundary conditions substituted for the distribution functions permits the regularity of these phenomena to be obtained.

The question of the influence of gas-molecule interaction with a surface, in fluxes caused by pressure and concentration inhomogeneities of a binary gas mixture in a capillary under arbitrary rarefaction, is examined from this viewpoint in this paper. The generalized thermodynamic fluxes are determined as functions of the Knudsen number Kn from the solution of a system of model BGK equations with Maxwell boundary conditions. In the viscous mode limit, the kinetic coefficients describing Poiseuille flow and mutual diffusion are independent of the details of gas-molecule interaction with the surface, while the cross coefficients (the barodiffusion constant and the diffusion-slip coefficient) are substantially governed by this interaction.

Let us examine the isothermal flow of a binary gas mixture in a long capillary of radius R. The density gradients of the first and second components are small and directed along the z axis. The mixture flow is described by the system of BGK equations

\[ \begin{align*}
    v_i \nabla f_i & = v_{ii} (M_i - f_i) + v_{ij} (M_{ij} - f_i) \\
    v_j \nabla f_j & = v_{jj} (M_j - f_j) + v_{jj} (M_{jj} - f_j)
\end{align*} \]

where \( M_i, M_{ij} \) are linear decompositions of the locally Maxwellian distribution functions, \( v_i (v_{iz}, v_{ir}, v_{ip}) \) is the molecule-velocity vector of the i-th component, \( v_{ii}, v_{ij} \) are the frequencies of the cross self-collisions, and \( f_i \) is the molecule-distribution function of the i-th component.

Let us take the boundary conditions for the distribution function as

\[ f_i (v_{iz}, v_{ir}, v_{ip}) = \epsilon_i M_i + (1 - \epsilon_i) f_i (v_{iz}, - v_{ir}, v_{ip}) \]

where \( \epsilon_i \) is the fraction of the molecules reflected diffusely from the wall.

Generalizing the results of [1, 2], the following system of integral equations for the velocities of the components can be obtained by integrating the system (1) along the characteristics and using (2):

\[ u_{i}(r) = \frac{1}{2\pi} \sum_{i} \left[ 1 - (1 - e_{i}) \exp \left( - \frac{v_{i}^{2}}{e_{i}} \right) \right]^{-1} \left[ 1 - e_{i} \right] \int_{0}^{r} \exp \left[ - \frac{v_{i}(r - r')}{e_{i}} \right] \Lambda_{i}(r') dr' \times \exp \left[ - \frac{v_{i}(l - r')}{e_{i}} \right] \Lambda_{i}(r') \frac{d\tau}{\tau} \exp \left[ - \frac{m_{i}r'^{2}}{2kT} \right] \, dv_{i}, d\psi \]

\[ \Lambda_{i}(r) = \frac{m_{i}}{kT} \left[ u_{i}(1 - x_{i}) + u_{i}a_{i} - \frac{k_{i}}{v_{i}} \frac{kT}{m_{i}} \right] \]

\[ a_{i} = \delta \frac{m_{i}}{m_{0}} \frac{v_{i} + v_{i}^{2}}{v_{i}^{2} + v_{i}}, \quad k_{i} = - \frac{1}{n_{i}} \frac{\partial n_{i}}{\partial z}, \quad m_{0} = m_{i} + m_{j}, \]

\[ r' = \left[ R^{2} + \xi^{2} - 2\xi r^{n} \right], \quad v_{i} = v_{i} + v_{j}^{n} \]

\[ l = 2\sqrt{R^{2} - r^{2}\sin^{2}p}, \quad \xi = 1/2 + r \cos p \]

Here \( m_{i} \) is the mass of the \( i \)-th component molecule, \( T \) is the absolute temperature, \( \delta \) is a cross-collision parameter (usually \( \delta = \gamma_{ij}' \)), and \( n_{i} \) is the density of the number of \( i \)-th component molecules.

An analogous equation is obtained for the \( j \)-th component by mutual replacement of the subscripts \( i \) \( \rightarrow \) \( j \).

The system (3) is solved by the Bubnov-Galerkin method with a quadratic trial function. It is convenient to represent the result of the solution as expressions for the thermodynamic kinetic coefficients which can be measured experimentally. According to the thermodynamics of irreversible processes, the generalized fluxes corresponding to the generalized forces \( X_{i} = \partial p/\partial z \) and \( X_{2} = \partial c_{i}/\partial z \) are written as follows:

\[ J_{i} = c_{i} \langle u_{i} \rangle_{x} + c_{i} \langle u_{j} \rangle_{x} = -L_{11}X_{1} - L_{12}X_{2} \]

\[ J_{j} = \langle u_{i} \rangle_{x} - \langle u_{j} \rangle_{x} = -L_{11}X_{1} - L_{12}X_{2} \]

where \( L_{11}, L_{12}, L_{21}, L_{22} \) are kinetic coefficients, \( \langle u_{i} \rangle_{x} \) is the mean velocity of the \( i \)-th component over the capillary cross section, and

\[ c_{i} = n_{i}/n, \quad n = n_{i} + n_{j}, \quad p = nkT \]

Solution of the system (3) permits finding the kinetic coefficients introduced according to (4) and (5):

\[ L_{11} = (n, m, v, a, \Delta)^{-1} \left[ -\pi [a_{i}a_{j}, (a_{i} - c_{i})^{2} + a_{i}b_{i}, (a_{i} - c_{i})^{2}] \right] \]

\[ + \pi^{2}a_{i}a_{j}(a_{i} - c_{i})\tau_{i}' + (a_{i} - c_{i})\tau_{j}' \right] + \frac{\pi^{2}}{12}a_{i}a_{j}(a_{i} - c_{i})^{2} + \frac{\pi^{2}}{12}a_{i}a_{j}(a_{i} - c_{i})^{2} \]

\[ \times \left[ a_{i}a_{j}(a_{i} - c_{i}) + a_{i}a_{j}(a_{i} - c_{i}) \right] \left[ a_{i}a_{j}(a_{i} - c_{i}) + a_{i}a_{j}(a_{i} - c_{i}) \right] \]

\[ L_{21} = L_{12} = (n, m, v, a, \Delta)^{-1} \left[ \pi^{2}a_{i}a_{j}(a_{i} - c_{i}) + \pi^{2}a_{i}a_{j}(a_{i} - c_{i}) \right] \]

\[ + \pi^{2}a_{i}a_{j}(a_{i} - c_{i})\tau_{i}' + (a_{i} - c_{i})\tau_{j}' \right] + \frac{\pi^{2}}{12}a_{i}a_{j}(a_{i} - c_{i})^{2} + \frac{\pi^{2}}{12}a_{i}a_{j}(a_{i} - c_{i})^{2} \]

\[ \times \left[ a_{i}a_{j}(a_{i} - c_{i}) + a_{i}a_{j}(a_{i} - c_{i}) \right] \left[ a_{i}a_{j}(a_{i} - c_{i}) + a_{i}a_{j}(a_{i} - c_{i}) \right] \]

\[ L_{22} = \left( \frac{1}{\delta \epsilon} - \frac{n^{2}a_{i}a_{j}}{\Delta} \right) \left[ (a_{i}a_{j}C_{2i} + b_{i}a_{j}C_{2j}) \right] \]

\[ \tau_{i} = C_{1i} - C_{1k} - \frac{1/4}{C_{1i}} \quad \tau_{j} = C_{1j} - C_{1k} \]

\[ \beta_{i} = (C_{1i}^{2} - C_{1j}^{2}), \quad \delta_{i} = \sqrt{\frac{m_{i}}{2kT}} \sin R \]

\[ \delta_{i} = (C_{1i}^{2} - C_{1j}^{2}), \quad \Delta = \frac{\pi^{2}a_{i}a_{j} + \pi^{2}a_{i}a_{j} + \pi^{2}a_{i}a_{j} + \pi^{2}a_{i}a_{j} + \pi^{2}a_{i}a_{j} + \pi^{2}a_{i}a_{j}}{12} \]

\[ C_{11} = \frac{1}{2} \sqrt{\frac{\pi}{2}} \frac{2 - e_{i}}{e_{i}} + \frac{1}{\delta_{i}^{2}} + \frac{8e_{i}^{2}k}{\delta_{i}^{4}} \]

\[ + \sum_{k=1}^{N} (1 - e_{k}) \left[ 8I_{1}(\delta_{k}) \right] \left[ e_{k}^{2}k - \frac{8}{3} e_{i} (2 - e_{i}) k^{2} + \frac{6e_{i}k}{\delta_{i}^{4}} \right] \]

\[ - \frac{2e_{i}k}{\delta_{i}^{4}} - \frac{6}{3} \frac{e_{i}k}{\delta_{i}^{4}} + \frac{2(2 - e_{i})k^{2}}{\delta_{i}^{4}} - \frac{8e_{i}^{2}k^{2}}{\delta_{i}^{4}} \]

\[ + \frac{8e_{i}k}{\delta_{i}^{4}} - \frac{8e_{i}k}{\delta_{i}^{4}} \left[ 16I_{3}(\delta_{k}) \right] \left[ 2e_{i}k^{2} - e_{i}^{2}(2 - e_{i}) k^{3} \right] \]

\[ + \frac{8e_{i}k}{\delta_{i}^{4}} - \frac{8e_{i}k}{\delta_{i}^{4}} \right] + 16I_{3}(\delta_{k}) \left[ 2e_{i}k^{2} - e_{i}^{2}(2 - e_{i}) k^{3} \right] \]

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