The problem of the development of secondary free-convection currents in forced turbulent flow in horizontal tubes for relatively weak thermal gravitation influence is analytically solved. The results of the solution are compared to experimental data.

Experimental data on local heat transfer [1] and on velocity and temperature profiles [2, 3] demonstrate that thermal gravitational forces exert a substantial influence on turbulent flow and heat exchange in horizontal tubes. Thermogravitational forces can affect the structure of the turbulence, which results in a variation in momentum and heat transfer and directly affects the averaged flow, which leads to the formation of secondary free-convection currents (as is the case in viscous-gravitational flow). Secondary free-convection currents for a turbulent flow may substantially differ from the pattern of secondary currents for viscous-gravitational flow in horizontal tubes due to high anisotropy and inhomogeneity of the momentum and heat transfer.

The boundaries and nature of the onset of the influence of thermogravitational forces on turbulent momentum and heat transfer have been examined [4] assuming that they do not directly influence the averaged flow. The threshold for the influence of thermogravitational forces on the velocity, temperature, frictional drag, and heat-transfer fields were clarified. The formation of secondary flows was not discussed in this article.

In this work the development of secondary free-convection flows in forced turbulent motion of an incompressible liquid in horizontal tubes will be discussed. The problem is solved by assuming that thermogravitational forces do not affect turbulent transfer. Conditions will be examined for a weak influence of thermogravitational forces, i.e., at relatively low Grashof numbers \( Gr = \beta g \varphi \nu d^4 / \lambda \nu^2 \).

By stating the problem this way we are able to clarify the contribution of mass forces for averaged motion and to approximately describe the flow at relatively low Gr.

We will write the kinetic equation for the eddy component as follows:

\[
\begin{align*}
\frac{\partial \omega_x}{\partial x} + \frac{u}{r} \frac{\partial \omega_x}{\partial \phi} + \frac{\partial \omega_y}{\partial r} + \frac{\partial \omega_z}{\partial \phi} + \frac{1}{r} \frac{\partial \omega_z}{\partial \phi} &= \omega_y \frac{\partial u}{\partial x} + \\
+ \omega_x \frac{\partial u}{\partial r} + \omega_y \frac{\partial u}{\partial \phi} + \frac{\partial u}{\partial r} \frac{\partial \omega_x}{\partial r} + \frac{\partial w}{\partial \phi} \frac{\partial \omega_x}{\partial \phi} + v \Delta \omega_x + \frac{\beta g}{r} \left( \frac{\partial (rt)}{\partial r} \sin \varphi + \frac{\partial t}{\partial \varphi} \cos \varphi \right) + \\
\omega_x &= \frac{1}{r} \left( \frac{\partial u}{\partial \phi} - \frac{\partial \varphi}{\partial \phi} \right), \\
\omega_z &= \frac{\partial \varphi}{\partial x} - \frac{\partial u}{\partial r} \\
\end{align*}
\]

where \( r \) is the current radius; \( x \) is the axial coordinate, counted off from the onset of heating; \( \varphi \) is an angle measured from the upper generatrix; \( t \) is temperature; \( u, v, \) and \( w \) are the axial, radial, and tangential components of the velocity, respectively; \( \nu \) is the kinematic viscosity coefficient; and \( \beta \) is the thermal-expansion coefficient.

The problem will be solved under the following assumptions. 
1. The process is steady-state. 
2. The physical properties of the liquid are constant except for a variation in density that can be taken into account in the mass force term. 
3. Flow is stabilized, i.e., the variation of all the hydrodynamic variables in the longitudinal coordinate are negligibly small. 
4. Molecular transfer is negligibly small in comparison with turbulent transfer. 
5. Turbulent vorticity transfer is represented in a gradient form, i.e., $v'w_x' = -\varepsilon \partial w_x / \partial r$. 
6. The turbulent vorticity transfer coefficient is equal to the turbulent momentum transfer coefficient and is described by Prandtl's dependence $\varepsilon / \nu = 0.4 \eta$. 
7. Prandtl's turbulence number $\operatorname{Pr}_T = 1.9$. The heat flux density on the wall is constant and the flow region beyond the onset of the heated section where $\partial t / \partial x = \text{const}$ is considered.

Equation (1) in linearized dimensionless form, taking into account the above assumptions, takes the form
\[
\frac{\partial}{\partial R} \left( RY \frac{\partial \Omega}{\partial R} \right) = \Gamma \frac{\partial (RT^+)}{\partial R} \sin \varphi \tag{2}
\]
where $\Gamma = 1.25 \text{Gr} / \text{Re} \text{Pr}^2$ is a small parameter,
\[
\Omega_0 = -\frac{\omega_0 d}{2a} = \frac{1}{R} \left[ \frac{\partial}{\partial R} (RW) - \frac{\partial V}{\partial \varphi} \right] \tag{3}
\]
$T_0^+ = (t_w - t_0) \rho \lambda / q_w$ is the initial temperature distribution characteristic for forced flow without the influence of thermogravitation; $R = 2r/d = 1 - \gamma$ is the current dimensionless radius; $d$ is the tube diameter; $v_w = \sqrt{\tau_w / \rho}$ is the velocity of friction; $V = \nu / u$ and $W = w / u$; $u$ is the mean rate of flow; $q_w$ is the heat flux density on the wall; $\text{Re} = \nu d / \nu$ is the Reynolds number, $\text{Pr}$ is Prandtl's number; $\rho$ is density; and $\lambda$ is specific heat.

Suppose the temperature profile $T_0^+$ is given by
\[
T_0^+ = 2.2 \ln \eta \times B (\text{Pr}) \tag{4}
\]
where $B(\text{Pr})$ is the Prandtl number function described according to data [5] by the expression
\[
B = 5 \ln \left( (5 \text{Pr} + 1) / 30 \right) + 8.55 + 5 \text{Pr} ;
\eta = v_w y / \nu \text{ is a dimensionless coordinate counted off from the wall.}

Solving Eqs. (2) and (4), we find an expression that describes the distribution of the eddy component:
\[
\Omega_x = -\Gamma \left\{ 1.1 (\ln Y)^3 + D \ln Y + C_1 \right\} \sin \varphi \tag{5}
\]
\[
D = 2.2 \ln \eta_0 + B (\text{Pr}) \nu = 4.5 \ln \nu + B (\text{Pr}) - 5, \quad \eta_0 = \nu_d / 2 \nu \text{.}
\]

The boundary condition has the form
\[
\frac{d \Omega_x}{d R} = 0, \quad R \to 0 \left\{ \begin{array}{c}
\varphi = \pi / 2
\end{array} \right.
\]

Using the definition of $\Omega_x$ from Eq. (4) and the continuity equation
\[
\partial (RV) / \partial R + \partial W / \partial \varphi = 0 \tag{6}
\]
we write down an equation for determining the tangential component of the velocity $W$:
\[
\frac{\partial}{\partial R} R \frac{\partial}{\partial R} (RW) + \frac{\partial W}{\partial R} = \frac{\partial}{\partial R} (R^2 \Omega_x) \tag{7}
\]
Representing the desired function $W$ in the form of a product of two functions,
\[
W = F (R) \sin \varphi \tag{8}
\]
the equation in partial derivatives of Eq. (7) is transformed into an ordinary differential equation,
\[
\frac{\partial^2 (RF)}{dR^2} + \frac{d (RF)}{dR} - \frac{(RF)}{R^2} = \frac{1}{R} \frac{d}{dR} (R^2 \Lambda) \tag{9}
\]
where $A = \Omega_x / \sin \varphi$. 