DYNAMICS OF SHOCK WAVES IN A LIQUID CONTAINING GAS Bubbles

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Results are presented of a numerical solution of the Korteweg-de Vries-Burgers equation that describes the propagation and establishment process for a stationary structure to a shock wave in a gas-liquid medium. Data are obtained on the time for the establishment of a stationary structure of a shock wave, propagation velocity, and amplitude oscillations in the front of the shock wave. Experiments are discussed on the basis of the results obtained for the study of shock waves in a liquid containing gas bubbles.

1. Compression waves of finite amplitude in gas-liquid media have been experimentally and theoretically investigated [1-12].

It has been shown [5, 8] that it is possible for a weak shock wave possessing oscillatory structure to form in a liquid with gas bubbles, given specified relationships between the effective mixture viscosity, intensity of the disturbance, and bubble radius. The case of strong shock waves in such a medium has also been considered [10].

The oscillatory structure of a standing shock wave was calculated in [6] and [8] on the basis of equations for a homogeneous single-velocity model assuming an adiabatic process within the suspension bubbles, and results have been presented [11] of a calculation of a standing wave based on a two-velocity model of the medium assuming a nonpolytropic process.

It has been demonstrated [4, 5] that the evolution of longwave disturbances in a liquid containing gas bubbles can be examined on the basis of the equations of the Korteweg-de Vries-Burgers equation, which is a model equation for describing the propagation process for waves of finite amplitude in a medium with weak dispersion and dissipation [13],

\[ u_t + u u_x - \eta \, u_{xx} + \beta \, u_{xxx} = 0 \] (1.1)

Here t is time, x is a coordinate, u is the velocity disturbance of the medium, \( \eta \) is the coefficient of effective viscosity of the medium, \( \beta = R_0^2 \, c_0 \, \alpha_0 / 6 \, \alpha_0 (1 - \alpha_0) \), \( R_0 \) is the diffusion length of a bubble, \( c_0 = \sqrt{p/\rho} \, (1 - \alpha_0) \) is a low-frequency approximation of the speed of sound in the gas-liquid medium, \( \rho \) is liquid density, and \( \alpha_0 \) is the initial gas content of the mixture by volume.

Equation (1.1) was written in a frame of reference moving at velocity \( c_0 \). When dissipation is due solely to viscous losses at the bubble-liquid boundary, the coefficient of effective viscosity of the mixture has the form

\[ \eta = 2 \, \nu / 3 \, \alpha_0 \] (1.2)

where \( \nu \) is the kinematic viscosity coefficient of the liquid. A remark regarding the coefficient in (1.2) can be found in [9]; that is, the actual dissipation coefficient in the mixture exceeds by a factor of 20 the coefficient calculated using Eq. (1.2). The coefficient \( \eta \) can be calculated more precisely preceding on the basis of results of [14]:

\[ \eta = \delta \, \omega \, R_0^2 / 6 \, \alpha_0 \, (1 - \alpha_0) \]
where \( \delta \) is the damping decrement, which may be represented in the form of a sum of decrements due to thermal dissipation, acoustic radiation losses, and losses calculated in [14]; \( \omega \) is the ripple frequency of a bubble, which for weak shock waves is close to the resonance frequency of the bubble calculated using the equilibrium values of pressure and radius.

The contribution to dissipation of losses caused by thermal effects and radiation may be substantial.

The stationary solutions of Eq. (1.1) of the form \( u = u(x - Vt) \), which describes the structure of the front, are obtained by integrating the ordinary differential equation

\[
\beta u'' - \eta u' + u'(u - V) = 0
\]

where \( V = \Delta u/2 \), \( \Delta u \) being the velocity discontinuity in the shock wave.

The existence criterion for shock waves with oscillatory structure, following Eq. (1.3), has the form [13]

\[
\eta (2 \beta \Delta u)^{-1} < 1
\]

The criterion (1.4) cannot hold in the course of the establishment of a standing front. The penetration probability for a shock wave with oscillatory structure into a gas-liquid mixture has been confirmed experimentally [4, 6-8, 10, 12].

It has been proposed [11] that times somewhat greater than were realized in these experiments are required for a shock wave to attain a standing structure.

In this work we shall investigate the process by which a shock wave is established and compare it to available experimental results.

2. The process by which a shock wave is established was studied on the basis of the decay problem for an arbitrary discontinuity by means of a numerical integration of Eq. (1.1).

The initial condition for Eq. (1.1) was selected in the form of a step function of the form

\[
\begin{align*}
  u(x, 0) &= u_0 \text{ m/sec}, \quad x \leq x_0 \\
  u(x, 0) &= u_0 \exp \left[-\left(x-x_0^2\right)/\epsilon^2\right], \quad x > x_0
\end{align*}
\]

The slope of the discontinuity front can be regulated by varying the parameter. The coordinates and form of the step function are connected by the broken line in Fig. 1. Moreover, numerical experiments with finite initial distributions of the form

\[
\begin{align*}
  u(x, 0) &= u_0 \exp \left[-\left(x-x_1^2\right)/\epsilon_1^2\right], \quad 0 \leq x \leq x_1 \\
  u(x, 0) &= u_0, \quad x_1 < x \leq x_0 \\
  u(x, 0) &= u_0 \exp \left[-\left(x-x_0^2\right)/\epsilon_2^2\right], \quad x > x_0
\end{align*}
\]

were carried out.

The initial disturbance (2.1) approximately corresponds to the conditions of previous experiments [8] and the distribution of the form (2.2), to other experiments [4, 6, 7], where the high-pressure chamber had limited volume.

The dispersion and dissipation coefficients were \( \beta = 10^{-4} \text{ m}^3/\text{sec} \) and \( \eta = 10^{-3} \text{ m}^2/\text{sec} \). These values of the coefficients roughly correspond to the conditions of previous experiments [4, 6-8] (\( u_0 = 1 \text{ m/sec}, \quad I = 0.08 \text{ m}, \quad l_1 = 0.04 \text{ m}, \quad \text{and} \quad l_1 = 0.4 \text{ m} \)).

The results of the calculation are presented in dimensional form to facilitate their comparison with the experimental results.

Equation (1.1) was approximated by an explicit three-layer finite-difference scheme of second order with respect to coordinate and time. The integration step was as follows: for coordinate, \( h = 0.01 \text{ m} \) and for time, \( \tau = 0.8 \cdot 10^{-3} \text{ sec} \).

Results are presented in Fig. 1 of a numerical integration of Eq. (1.1) with initial condition (2.1); the disturbance profile is depicted for \( t = 2 \text{ sec} \). A standing shock wave with oscillatory structure was estab-