Studies on the effect of electromagnetic body forces on the hydrodynamic pattern of the flow around bodies propelled by internal sources of electromagnetic fields have become of interest in connection with the design of magnetohydrodynamic propellers for submarines and surface vessels (see, e.g., [1, 2] and their bibliographies). Such studies for the case of a sphere as an example were started in [3-5] for fixed sources, which were chosen from qualitative considerations and are not optimum.

Clearly, the distributions of the electric and magnetic potentials at the surface of the sphere should be optimum, ensuring that the electrical energy consumption for propulsion at a given speed be minimum. Our goal here is to formulate the complete variational problem for determining the optimum potentials, construct the solutions of some simplified (variational) problems, and analyze them.

1. We consider a sphere of radius $a$ with an internal source of fields, which was described in [3]. Electromagnetic fields in a liquid are characterized by the scalar potentials

$$
E = -\nabla [\phi(r, \theta) \sin m\alpha], \quad B = -\nabla [\chi(r, \theta) \cos m\alpha].
$$

The velocity field is assumed to be axisymmetric,

$$
\mathbf{v} = \frac{1}{r \sin \theta} \left( -\frac{1}{r} \frac{\partial \phi}{\partial \theta} \mathbf{e}_r + \frac{\partial \phi}{\partial \theta} \mathbf{e}_\theta \right),
$$

and is described by the stream function $\psi(r, \theta)$ (the sense of the axisymmetry assumption and some considerations concerning its applicability are given in [3]). The functions $\phi(r, \theta)$, $\chi(r, \theta)$, $\psi(r, \theta)$, and $w(r, \theta)$ [vorticity $\text{curl} \, \mathbf{v} = \omega (r, \theta) \mathbf{e}_\theta$] are determined from the problem

$$
L \psi = -\frac{m \chi}{r \sin \theta} w, \quad LX = 0
$$

$$
\left( L = \frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \frac{\partial}{\partial \theta} - \frac{m^2}{r^2 \sin^2 \theta} \right); \\
-\frac{1}{2r} \left[ \frac{\partial \psi}{\partial \theta} \frac{\partial w}{\partial \theta \sin \theta} - \frac{\partial \chi}{\partial \theta} \frac{\partial w}{\partial r \sin \theta} \right] + \frac{1}{r \sin \theta} E^2 (r \sin \theta \omega) + N \langle \text{curl} \, \mathbf{F} \rangle = 0;
$$

$$
E^2 \psi - rw \sin \theta = 0 \left( E^2 = \frac{\partial^2}{\partial r^2} + \frac{\sin \theta}{r^2} \frac{\partial}{\partial \theta} \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \right);
$$

The undetermined functions $\chi_0(\theta)$ and $\psi_0(\theta)$ in the boundary conditions (1.6) should be found by solving the variational problem of minimizing the functional representing the electric power consumption

$$f_\mu \sigma^2 \int_\Omega \int_\Sigma (\mathbf{j} \cdot \mathbf{E}) d\omega$$

(integral over the space outside the sphere). The problem is simplified if $Q = \int_\Sigma \int_\Sigma (\mathbf{j} \cdot \mathbf{E}) d\omega$ is written as an integral over the surface of the sphere. In the general case (when $\mathbf{j}$ is expressed in terms of curl $\mathbf{H}$) this transition is made by using the Poynting vector; a corresponding representation is also possible in the induction-less approximation under consideration since $\mathbf{j} \cdot \mathbf{E} = -\text{div}(\mathbf{E}) = -\text{div} \phi [\mathbf{E} + \mathbf{v} \times \mathbf{B}]$ because $\text{div} \mathbf{j} = 0$. Hence

$$Q = \int_0^\pi \psi(1, \theta) \left[ -\frac{\partial \psi(1, \theta)}{\partial r} + \frac{m \chi(1, \theta) \sin \theta}{\sin \theta} \psi_0(1, \theta) \right] \sin \theta d\theta. \quad (1.7)$$

(The conditions of sticking on the sphere have not yet been taken into account so that this form of the functional $Q$ could also be used for an ideal liquid.)

The sphere under consideration is self-propelled. This means that zero electromagnetic tractive force acts on the sphere. This condition can be written as

$$8N \int_0^\pi \left\{ \left\langle f_r(r, \theta) \right\rangle \cos \theta - \left\langle f_\theta(r, \theta) \right\rangle \sin \theta \right\} r^2 \sin \theta d\theta dr +$$

$$+ \int_0^\pi \left\{ \frac{4}{16} \left[ \psi(1, \theta) - \frac{\partial \psi(1, \theta)}{\partial r} \right] - 4N \left\langle \phi_0(1, \theta) \right\rangle \right\} \sin^2 \theta d\theta = 0. \quad (1.8)$$

The forces $\left\langle f_r \right\rangle$ and $\left\langle f_\theta \right\rangle$ (the angular brackets denote averaging over the angle $\alpha$) are expressed in terms of the functions $\psi_0, \chi, \psi$ and their derivatives from formulas in [6].

The variational problem under study thus is a problem for an angular extremum and consists of choosing the functions $\chi, \psi, \psi$ that would ensure the minimum of the functional $Q (1.7)$ with the auxiliary conditions (1.3)-(1.6), the condition of self-propulsion (1.7), and a condition limiting the scale of the magnetic field. It is most desirable from the physical standpoint to formulate the last condition as the requirement that the maximum value of the dimensionless magnetic induction at the surface of the sphere be equal to 1,

$$\left| B \right|_{\text{max}}|_{r=1} = 1 \quad (1.9a)$$

(the $B_0$ used when the scale is made dimensionless is the maximum magnetic induction at the surface). The simple limitation

$$\left( B_0 \right)_{\text{max}}|_{r=1} = \left( \frac{m \chi(1, \theta)}{\sin \theta} \right)_{\text{max}} = 1 \quad (1.9b)$$

($B_0$ is the maximum value of $B_0$) is meaningful. Such a limitation cannot be easily implemented directly in a variational problem. The imposition of a limitation on the functional of $\chi(1, \theta)$ is more suited to the nature of the variational problem. Here we consider limitations of the type

$$\int_0^\pi h_1(\theta) \chi(1, \theta) d\theta = \text{const} \quad (1.10a)$$

or

$$\int_0^\pi h_2(\theta) \chi^2(1, \theta) d\theta = \text{const} \quad (1.10b)$$