STRUCTURE OF A SHOCK WAVE IN WHICH MULTIPLE
IONIZATION OF THE ATOMS IS TAKING PLACE

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Ionization relaxation in a shock wave of very large amplitude is considered, the atoms behind the front of the shock wave being multiply ionized. In calculating the structure of the shock wave and the kinetics of ionization, allowance is made for the electron component of the thermal conductivity which plays an important role in this. A simplified method of calculating the kinetics of multiple ionization is proposed, and an application of this method is presented. The results of the structure calculation show that, as a result of heating by thermal conduction, the gas is considerably ionized even in front of the jump in compression, while the electron component of the thermal conductivity passes through a maximum.

Ionization relaxation behind the leading edge of a shock wave in a gas has been studied both theoretically and experimentally; however, only relatively weak shock waves have been considered, the ionization of the atoms in these being correspondingly weak.* One of the main points of interest is the mechanism of primary ionization, involving the "seed" electrons from which the electron avalanche begins, subsequently developing by way of the ionization of the atoms under the impact of the electrons. In the majority of publications, it has been considered that the primary electrons appear by virtue of atom-atom collisions, although the possibility of initial ionization by virtue of light irradiation has also been considered. An important feature in the process of ionization relaxation is the temperature difference between the electrons and the heavy particles (atoms and ions). In the shock wave, the kinetic energy of the incident flow is converted into thermal energy of the heavy particles; this energy is gradually transferred from the atoms and ions to the electrons, which ionize the atoms. Owing to the retarded exchange of energy between the heavy and light particles, the electron temperature lags behind that of the atoms.

The structure of a shock wave in a partly or completely ionized gas has also been calculated on the assumption of a constant degree of ionization. Such calculations reveal the importance of the electron component of the thermal conductivity, as a result of which the electron gas in front of the jump in compression is heated to a temperature very similar to the equilibrium temperature behind the shock wave. Calculations show that, in severe ionization, the electron thermal conductivity and the exchange of energy between the electrons and ions exert a comparable influence on the structure of the relaxation layer in the shock wave. We note that, in considering the kinetics of ionization, the electron thermal conductivity has never yet been taken into account, possibly because it is not particularly important in weak ionization.

In this paper we shall consider the structure of very strong shock waves with an equilibrium temperature of the order of hundreds of thousands of degrees and over, in which the atoms are multiply ionized. The kinetics of multiple ionization were considered earlier by V. A. Bronshtën and A. N. Chigorin [3] under conditions similar to those existing in shock waves, in connection with the problem of the motion of high-speed meteors through the atmosphere [4, 5]. However, these calculations, although very comprehensive and apparently the only ones of their kind, still fail to provide a complete picture of the real structure

*An exposition of this problem may be found, for example, in the book of Zel'dovich and Raizer [1]; there is a fairly complete bibliography in a later publication [2].
of the shock wave, in view of the fact that they omit all consideration of the electron thermal conductivity. As we shall see, this component of thermal conductivity plays a vital role in establishing the electron temperature distribution, which in turn determines the rate of ionization. Hydrodynamic aspects were also omitted from the calculation; the volume of gas was considered constant, and the electron and atomic temperatures were balanced under the assumption of a constant store of total internal energy. (In the subsequent analysis we shall also reformulate the problem of the ionization reaction velocities for ions of different multiplicities and solve it in a novel manner.)

It should be mentioned that the problem here considered differs very greatly in the mathematical respect from the shock-wave structure problems solved earlier. The problem is reduced, in the usual way, to one of finding an integrated curve relating the singular points of the system, the states in the neighborhood of equilibrium and the establishment of equilibrium itself are determined by the behavior of the integrated curve of the system in the neighborhood of the singular points. However, in contrast to the problems treated earlier [6, 7], the kinetics of ionization and the electron thermal conductivity are taken into account at the same time; the result is that the behavior of the curve in the neighborhood of a singular point has to be analyzed, not in a plane, but in phase space, and this greatly complicates both the qualitative analysis itself and the numerical calculations. Whereas numerical integration in a plane in the neighborhood of a saddle point presents no serious difficulties, in view of the specific character of the saddle, in the three-dimensional case matters are not so simple: A singular point of the generalized-saddle type constitutes an ordinary saddle in some planes and a node in others, so that a small error in numerical integration may lead to a serious departure from the desired solution.

1. Let us proceed to formulate the problem. Let us consider a plane shock wave in a coordinate system linked to the wave, i.e., the one-dimensional steady motion of the gas in infinite space. In front of the wave the gas is, generally speaking, cold and unionized; behind the wave the atoms are multiply ionized because of the high temperature. The thermodynamical-equilibrium state of the gas behind the wave is completely determined by the initial density and by a certain parameter characterizing the amplitude of the wave, for example, the velocity of the leading edge or the temperature behind it. This state (degree of ionization, density, etc.) may be calculated from the existing relationships governing shock rupture effects and from the thermodynamical functions of the ionized gas [1, 8].

We shall neglect the viscosity and thermal conductivity of the heavy particles, replacing the viscous layer by a break. The viscosity of the electron gas is also unimportant, but the electron thermal conductivity is much greater than that of the ions and atoms, and acts both in front of and behind the jump in compression. The temperatures of the electron and atomic-ionic gas are regarded as different.

The gas is considered as being monotonic, since in such a strong wave (one involving multiple ionization) the molecules (if the cold gas is molecular) dissociate extremely rapidly and at relatively low temperatures.

Radiation and radiative heat transfer are not taken into account [1]. If the density of the gas is high, let us suppose of the order of the density of atmospheric air, the range of the radiation, determining the spatial scale of the region of radiant heat transfer, is far greater than the zones of ionization relaxation and electron heating in front of the leading edge, which are the only interesting features in the problem under consideration. If, however, the gas is rarefied, the heated region is transparent to the radiation, the density of the radiation within it is far lower than the thermodynamic-equilibrium value, and the radiation has very little effect on the structure of the shock wave.

The question as to the primary ionization of the cold gas does not arise at all in the case of a strong shock wave, since for temperatures of the heated region equal to hundreds of thousands of degrees the radiation is always strong enough to create a small number of electrons in front of the wave, and these initiate the electron avalanche. The structure of the shock wave is almost independent of the extent of the initial ionization.

Let us introduce some notation and definitions. In every state, ions of various multiplicities m are contained in the gas. All the heavy particles will be called ions for brevity, atoms being treated as ions of zero charge m = 0.

*The rupture relationships may be derived from the system of equations describing the structure of the wave by extending the spatial coordinate to infinity in both directions.