A method of calculating effective masses of multirow clusters of elastic-cylindrical rods is presented. The flow of liquid caused by lateral vibrations of the rods is described approximately using a model of cells.

1. The Model of the Cells. The work [1] is devoted to calculation of effective masses of single-row clusters of cylindrical rods. As far as calculation of effective masses of multirow clusters is concerned, this has been very little studied up to now because of the complexity of taking into account the restriction of the flow of liquid in the cluster.

The proposed method is based on replacing the multirow cluster of cylindrical rods (Fig. 1a) by a combination of isolated cells, one of which is shown in Fig. 1b. The cell consists of two coaxial cylinders; the inner cylinder represents a rod of the cluster, and the outer cylinder simulates the restriction of the flow which is caused by lateral vibrations of the inner cylinder.

Let each rod of the cluster be surrounded by a liquid which fills a certain region (Fig. 1c). It is then possible to find from the condition of continuity of flow of liquid flowing through the cell and through the above mentioned region a relationship between the radius of the outer cylinder b and the spacing h of the rods in the cluster, and also the connection between the relationship between the radii of the cylinders forming the cells and the density of the cluster

\[ b = \pi^{-1/2}h, \quad \frac{a}{b} = \frac{1}{2\sqrt{\pi}}q \quad (q = \frac{2a}{h}) \]  

(1.1)

where \( a \) is the outer radius of the rod of the cluster, and \( q \) is the density of the cluster.

Hence, determination of the effective mass of a rod located in the cluster is reduced to calculation of the effective mass of this rod located inside a cylinder with a radius b.

The problem is solved by the hypothesis that the rods of the cluster have a finite length and arbitrary supports at the ends, and they carry out small elastic vibrations.

The liquid which flows round the rod is considered as ideal and compressible, but its flow is considered as irrotational.

The flow of liquid in the cell is described by the wave equation

\[ \frac{\partial^2 \varphi}{\partial t^2} + \frac{1}{r} \frac{\partial \varphi}{\partial r} - \frac{1}{r^2} \frac{\partial^2 \varphi}{\partial \theta^2} + \frac{\partial^2 \varphi}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 \varphi}{\partial t^2} = 0 \]  

(1.2)
with the following boundary conditions:

\[
\frac{\partial \varphi}{\partial r} \bigg|_{r=a} = \frac{\partial x}{\partial t} \cos \theta, \quad \frac{\partial \varphi}{\partial r} \bigg|_{r=b} = 0 \\
\frac{\partial \varphi}{\partial z} \bigg|_{z=0} = 0, \quad \frac{\partial \varphi}{\partial z} \bigg|_{z=l} = 0
\]  

(1.3)

Here \( \varphi \) is the velocity potential, \( l \) is the length of the rod, and \( x \) is the displacement of the rod which is equal to

\[x = \sum_{n=1}^{\infty} q_n(t) f_n(z), \quad q_n = C_1 \cos p_n^0 t + C_2 \sin p_n^0 t \]  

(1.4)

In these expressions \( f_n \) is the form of the \( n \)-th mode of the vibrations of the rod in the liquid, which has been taken to be equal to the mode of the vibrations in a vacuum; \( q_n \) is the main coordinate; \( C_1 \) and \( C_2 \) are the arbitrary constants determined by the initial conditions of the problem; \( p_n^0 \) is the frequency of the \( n \)-th mode of the free vibrations of the rod in the liquid.

The velocity potential is determined in the following form:

\[\varphi = \cos \theta \sum_{n=1}^{\infty} q_n^* \sum_{s=1}^{\infty} F_s(r) N_s(z) \]  

(1.5)

The function \( F_s(r) \) must satisfy the first two boundary conditions (1.3), and the function \( N_s(z) \) must satisfy the remaining boundary conditions. The two latter conditions are satisfied if we assume

\[N_s(z) = \cos \left( \frac{\pi z}{l} \right) \]  

(1.6)

By substituting the expression (1.6) into the original Eqs. (1.2), taking (1.6) into account, we have

\[\frac{d^2 F_s}{dr^2} + \frac{1}{r} \frac{d F_s}{dr} - \left[ \frac{\pi s^2}{l^2} + \frac{1}{r} \left( \frac{p_n^0}{c} \right)^2 \right] F_s = 0 \]  

(1.7)

The solution of this equation will be

\[F_s = A I_1 \left[ \frac{\pi s}{l} + \frac{p_n^0}{c} \right] r + B K_1 \left[ \frac{\pi s}{l} + \frac{p_n^0}{c} \right] r \]  

(1.8)

Here \( A \) and \( B \) are arbitrary constants; \( I_1 \) and \( K_1 \) are modified Bessel functions.

After calculation of the arbitrary constants [using the first two conditions \( \varphi \), (1.3)], the formula for the velocity potential assumes the form

\[\varphi = \frac{2}{\pi} \cos \theta \sum_{n=1}^{\infty} q_n^* \sum_{s=1}^{\infty} \left[ K_1(\alpha r) - \frac{K_1^{(ab)}}{I_1^{(ab)}} f_s(\alpha r) \right] x \left[ K_1^{(ab)}(\alpha a) - \frac{K_1^{(ab)}}{I_1^{(ab)}} f_s^{(ab)}(\alpha a) \right]^{1/2} \cos \frac{\pi s}{l} \int_0^l f_s(z) \cos \frac{\pi s}{l} dz \]  

(1.9)

The kinetic energy of the liquid which fills the doubly connected region is equal to

\[T = -\frac{\rho^0}{2} \sum_{S_1} \sum_{n=1}^{\infty} q_n^* \sum_{s=1}^{\infty} \left[ \frac{\pi s}{l} \int_0^l f_s(z) \cos \frac{\pi s}{l} dz \right]^2 \]  

(1.10)

Here \( S_1 \) is the surface of the rod, and \( \rho^0 \) is the density of the liquid.

A similar integral over the surface of the outer cylinder of the cell in the same formula disappears due to the second condition (1.3).

After substitution of the value of the velocity potential into this formula and integration, which is carried out over the surface of the rod, the kinetic energy of the liquid is determined

\[T = \rho^0 \sum_{n=1}^{\infty} q_n^* \sum_{s=1}^{\infty} \frac{\pi s}{l} \int_0^l f_s(z) \cos \frac{\pi s}{l} dz \]  

(1.11)

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