THE QUASISTATIONARY ONE-DIMENSIONAL MOTION
OF AERATED OIL AND GAS

Yu. A. Afinogenov

The problem of the one-dimensional motion of aerated oil and gas arose in connection with
the design of a development of the Samotlorsk oilfield, level A of which is a single massive
stratum with a gas cap in the upper part and ground water in the underlying part. It is as-
sumed that the area of the field is covered with a network of wells at a given spacing. The
well filters are located arbitrarily in a single area which is called the plane of oil sampling.
The position of the gas-oil contact and the water-oil contact are considered as functions of
the time in the sampling plane in order to determine the depletion of the stock of oil and
also to locate the sampling plane most advantageously in depth.

The problem falls into two parts: the motion of the gas-oil contact and the plane in which the oil is
subject to saturation pressure by the gas; and the motion of the water-oil contact and the plane of satu-
ration pressure. We take the sampling plane as the plane from which measurements are made. The x axis
is taken as the axis of the change in the positions of the gas-oil contacts. Let L denote the distance from
the sampling plane in the upper boundary of the gas cap: s₁ is the coordinate of the position of the plane of
pressure saturation in the problem of the motion of the gas-oil contact; s₂ is the coordinate of the position of
the gas-oil contact; s₃ is the coordinate of the position of the plane saturation in the problem of the
motion of the water-oil contact; s₄ is the coordinate of the position of the water-oil contact.

We divide s₁, s₂, s₃ and s₄ by L and adopt the nondimensional coordinates u₁, u₂, u₃ and u₄; L₁ and L₂
are the nondimensional coordinates of the initial position of the gas-oil contact and the water-oil contact,
respectively.

Suppose that at the initial moment of time the pressure in the stratum is pᵣ > pᵣ, where pᵣ is the sat-
uration pressure under which dissolved gas is separated out from the oil. We assume that in the process
of developing the stratum the pressure in the sampling plane is fixed at a constant value and is equal to
pₙ < pᵣ.

In addition we assume the following:

1) the generalized Darcy law holds for the oil pressure occupying the proportion σ of unit volume
   of the interstitial space [f(σ) is the relative permeability of the oil]

\[ v = - \frac{k}{μ} f(σ) \frac{∂p}{∂x} \]  \hspace{1cm} (1)

2) the oil density ρ is effectively constant; ρ = const (it is independent of the quantity of extracted
gas);

3) gas extraction occurs instantaneously and depends on the pressure drop below the saturation pres-
sure

\[ σ = 1 - a (p₁ - p) \] \hspace{1cm} (2)

where a is a small quantity of dimension atm⁻¹;

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 63-
4) the region occupied by the gas, \( x \equiv (s_1, L) \), is remote from the region of strong pressure perturbations so that, assuming the process is isothermal, we can write
\[
p*(L - s_1) = p*(L - l)
\]
where \( I = s_1(0) \) is the initial boundary between the oil and the gas, \( p* \) is the variable pressure in the region of the gas as it expands;

5) in the zone \( x \equiv [s_1(t), s_2(t)] \) the motion is rigid, so that
\[
p = a_1 x + b_1
\]
where \( a_1, b_1 \) are determined from the boundary conditions
\[
x = s_1, \quad p = p_1, \quad x = s_2, \quad p = p*
\]
From this
\[
p = p^*(L - l)/(L - s_1) - p_1 (x - s_1) + p_1
\]
The kinematic condition for \( x = s_2 \) yields
\[
m \frac{ds_2}{dt} = \frac{k \cdot \frac{\partial p}{\partial x}}{\mu} = \frac{k \cdot \frac{p^*(L - l)/(L - s_1) - p_1}{s_2 - s_1}}
\]
where \( m \) is the porosity of the stratum.

Equation (5) has to be integrated subject to the initial condition
\[
s_2(0) = l
\]
6) in the zone \( x \equiv (0, s_1) \) the oil moves with the gas.

By the Slikhter-Kozena equation, \( k = Am^3/(1 - m)^2 \), where \( A \) is a constant depending on the grain size of the skeleton of the porous medium. Replacing \( m \) by \( m^c \), we have
\[
k(\xi) = A \frac{m^3 \sigma^3}{(1 - m^c)^2}
\]
and for the phase permeability [1]
\[
f(\xi) = \frac{k(\xi)}{k} = \frac{(1 - m^c)^2}{(1 - m)^2} \sigma^3
\]
The combination of the law of mass conservation for the oil and Darcy's law yields
\[
\frac{k \cdot \frac{\partial \sigma}{\partial x}}{\mu} \left[ \frac{(1 - m^c)^3}{(1 - m)^3} \sigma^3 \frac{\partial \sigma}{\partial x} \right] = \frac{m \sigma}{\partial t}
\]
If we substitute (2) for \( \sigma \) in this equation and simplify, putting \( 1 - a(p_1 - p) \sigma = 1 - na (p_1 - p) \) \((n = 2, 3)\), we obtain
\[
\frac{k \cdot \frac{\partial \sigma}{\partial x}}{\mu} \left[ \frac{(1 - m)}{1 - m} a (p_1 - p) \frac{\partial \sigma}{\partial \xi} \right] = \frac{\partial \sigma}{\partial t}
\]
Let \( q \) denote the pressure of the rock bed lying above the roof of the stratum. We can reduce Eq. (7) to nondimensional form by putting
\[
\frac{\xi}{L} = \frac{\xi_0}{L}, \quad \frac{\xi_1}{L} = \frac{\xi_1}{L}, \quad \frac{\xi_2}{L} = \frac{\xi_2}{L}, \quad \frac{\xi}{L} = \frac{\xi}{L}, \quad \tau = \frac{kt}{\mu \nu L^2}
\]
\[
(x^2 + \beta) \frac{\partial \xi}{\partial x^2} + \frac{\partial \xi}{\partial x} = \frac{\partial \xi}{\partial \tau}, \quad x = \frac{(3 - m^c) \xi_2}{1 - m}, \quad \beta = 1 - x^2_1
\]
Equation (8) has to be integrated in the region \( y \equiv (0, u_1), \tau > 0 \) under the conditions
\[
\xi(0, \tau) = \xi_0, \quad \xi(u_1, \tau) = \xi_1, \quad u_1(0) = 0
\]