PROPAGATION OF WEAK PERTURBATIONS IN TWO-PHASE MEDIA WITH PHASE TRANSITIONS

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The linearized equations of fluid mechanics [1] and the equation of state of the phases are used to investigate the propagation of weak perturbations in two-phase media which are a mixture of a gas and drops or particles. Allowance is made for possible phase transitions. The dependences of the wave vector on the perturbation frequency are obtained. An estimate is made of the effect of mass exchange between the phases on the nature of the dispersion relations. Some theoretical and experimental investigations devoted to the propagation of sound in two-phase media have been made, for example, in [2-5]. Throughout the paper the quantities that refer to the gas and the particles carry the subscripts 1 and 2, respectively. The subscript 0 refers to the unperturbed state; the subscript 3 to the saturation state.

1. The equations of conservation of mass, momentum, and energy of the phases [1] take the following form after linearization:

\[
\begin{align*}
\frac{\partial p_1}{\partial t} + \rho_1 \text{div} \mathbf{v}_1 &= J^0 - J_0, \\
\frac{\partial \rho_2}{\partial t} + \rho_2 \text{div} \mathbf{v}_2 &= J_0 - J^0, \\
\rho_1 \frac{\partial v_1}{\partial t} &= -\alpha \nabla p - \rho_2 \mathbf{f}, \\
\rho_2 \frac{\partial v_2}{\partial t} &= -(1 - \alpha) \nabla p + \rho_2 \mathbf{f}.
\end{align*}
\]

(1.1)

Here \(\rho\), \(p\), \(i\), and \(e\) are the perturbations of the mean density, pressure, enthalpy, and internal energy; \(\alpha\) is the volume concentration of the gas; \(l\) is the specific heat of vaporization; and \(J_0\) and \(J^0\) are, respectively, the observed rates of condensation and vaporization in unit volume of the mixture.

If the relations are given that reflect the force interaction \(f\), the heat exchange \(q\), and the mass exchange \(J_0\) and \(J^0\), and if the equations of state of the phases are also given, the system (1.1) is closed. For \(f\) and \(q\), we can take the following linear relations, which are valid for laminar flow around an isolated sphere (for sufficiently small numbers \(N_Re\) of the relative flow):

\[
\begin{align*}
f &= \frac{48 \eta_1}{\rho_1 c_1^2} (v_1 - v_2), \\
q &= \frac{12 k_1}{\rho_1 c_1^2} (T_1 - T_2)
\end{align*}
\]

(1.2)

Here \(\eta_1\) is the viscosity; \(k_1\) is the coefficient of heat transfer of the material of the first phase; \(\rho_2^0\) is the density of the material of the drops (particles) and \(d\) is their diameter.

The effect of the volume of the particles on the frictional force between the phases in the region of small \(N_Re\) can be taken into account by means of an additional factor [6, 7]:

\[
\varphi = (1 / \alpha)^{3/5}
\]

(1.3)

The equations of the kinetics of the phase transitions when there are small superheatings or supercoolings \((T_1 - T_3)/T_3\) can be written in the form [1]

\[
\begin{align*}
J_0 &= \frac{6 (1 - \alpha) l}{T_0 d}, \quad F_0 (T_3 - T_1), \\
J^0 &= \frac{6 (1 - \alpha) l}{T_0 d} F^0 (T_3 - T_2)
\end{align*}
\]

(1.4)
where $F_0$ and $F^*$ are coefficients determined experimentally or by other considerations.

Note that these equations can be used if the linear dimension of the perturbations (the wavelength) is much greater than the drop diameter and the perturbation amplitudes are sufficiently small.

2. For a cone-component two-phase medium we introduce the following equations of state of the phases:

$$p = \rho_1 R_p T_1, \quad i_1 (p, T) = c_2 (T_1 - T^0) + p / \rho_2 + I (p) + c_2 (T_1 - T_1)^2$$

$$\rho_2 = \text{const}, \quad i_2 (p, T) = c_2 (T_1 - T^0) + p / \rho_2$$

We introduce the dimensionless variables

$$P = \frac{p}{p_0}, \quad U = \frac{\rho}{\rho_0}, \quad \Theta = \frac{T}{T_0}, \quad \Phi = \frac{\rho}{p_0} \left( \frac{a^*}{\rho_0} \right)$$

and the parameters

$$C_1 = \frac{c_1}{T_0}, \quad C_2 = \frac{c_2}{T_0}, \quad \theta_0 = \frac{T_1}{T_0}, \quad L = \frac{L_0}{a^*}, \quad r = \frac{\rho_0}{\rho_2}$$

and also the reduced variables

$$f^* = \frac{\rho_0 f}{\rho_2 a^*}, \quad q^* = \frac{\rho_0 q}{\rho_2 a^*}, \quad J^* = \frac{J_0}{\rho_2 a^*}, \quad J_0 = \frac{J_0}{\rho_2 a^*}, \quad \tau = \frac{a t}{a^*}$$

Here $a$ is the velocity of sound in the first phase and $\tau$ is the reduced time in units of length.

Linearizing (2.1) and using (1.1)-(1.4) and (2.2)-(2.4), we obtain the following system of equations (one-dimensional case):

$$\frac{\partial \Phi}{\partial \tau} + a^* \frac{\partial U}{\partial x} = J^* - J_0, \quad \frac{\partial \Phi}{\partial \tau} + \frac{1}{2} \frac{\partial J^*}{\partial \tau} = J_0$$

$$\frac{\partial U}{\partial \tau} = \frac{1}{2} \frac{\partial J^*}{\partial \tau} + \frac{1}{2} \frac{\partial \Phi}{\partial x} + f^* = 0, \quad \frac{\partial \Phi}{\partial \tau} + \frac{1}{2} \frac{\partial J^*}{\partial \tau} = J_0$$

$$\frac{\partial \theta_0}{\partial \tau} + (1 - a) C_2 \frac{\partial \theta_0}{\partial \tau} - \frac{1}{2} \frac{\partial \Phi}{\partial \tau} = (J_0 - J^*) L_0$$

Here and in what follows, the primes denote the total derivatives with respect to the dimensionless pressure; $\tau_T, \tau_T^0, \tau_T^2$, and $\tau_0$ are reduced relaxation times (with the dimensions of a length).

Let us consider the propagation of plane periodic waves in a medium described by the equations (2.5); we shall seek the solutions of this system in the form of a damped traveling wave $\exp[i(kx - \omega t)]$. The condition for the existence of a nontrivial solution of this type leads to the following relationship between the wave vector and the dimensionless perturbation frequency $\eta (\eta = \omega \tau_T / a)$:

$$K^2 = \frac{1}{\tau_T} \left( \frac{1 + m - \eta \nu' \nu'_{\tau_T}}{1 + \frac{\Pi_0 + n_0 \Pi_0 - \Pi_0 \Pi_0}{\Pi_0 \Pi_0 - n_0 \Pi_0}} \right)$$

$$m = (1 - a) / \nu_{T}, \quad m^* = 1 - a (r - 1), \quad \Pi_0 = \text{Re} \Pi_0 + \text{Im} \Pi_0$$

$$\text{Re} \Pi_0 = (1 - r) \left[ \theta' (C_1 + m C_2) + \beta - \frac{1}{2} \right] \left( 1 - a \right) \left( \frac{1}{\nu_0} + \frac{1}{\nu_T} \right) - \left( 1 - a \right) C_2 \frac{\beta \nu' \nu'_{\tau_T} \nu' \nu'_{\tau_T}}{\tau_T} + \frac{1}{\tau_T} - L$$

$$\text{Im} \Pi_0 = \eta (r - 1) / (1 - r) \left[ \theta' C_1 + \gamma L' \frac{1}{\tau_T} \right]$$

$$\Pi_1 = (1 - a) \theta' \left( \frac{1}{\nu_0} + \frac{1}{\nu_T} \right) - \eta \left( \frac{1}{\nu_0} - \beta \right) \frac{a^*}{\tau_T}, \quad \Pi_2 = (1 - a) \frac{L}{\tau_T} - \eta \frac{a^* C_1}{\tau_T}$$

$$\gamma = \theta' \left( \frac{L}{C_2} \frac{1}{\tau_T} - \eta \frac{C_2}{\tau_T} \right), \quad \Pi_4 = 1 + \frac{\tau_T}{C_2} - \eta$$

$$\eta = \frac{\theta' \nu'_{\tau_T} \left( \frac{L}{C_2} + \frac{1 - r}{\tau_T} \right)}{C_2 \tau_T}$$