ONE-DIMENSIONAL DIFFUSIONAL MODEL
OF HETEROGENEOUS (GAS–FILM) DETONATION

S. A. Lesnyak and V. G. Slutskii

The detonation process in a tube filled with a gaseous oxidant (oxygen) and which has a "thick" layer of fuel (carbon with a low vapor pressure) deposited over its entire perimeter is examined; the weight ratio of fuel to oxidant considerably exceeds the stoichiometric ratio. It is assumed that the rate of heat release is determined by the diffusional (noninstantaneous) process of mixing of the vaporizing fuel and the oxidant. An estimate is made of the effect on the detonation parameters of heterogeneity in the composition over the cross section of the tube and of friction and heat losses. Dependences of the detonation parameters (propagation velocity, pressure profile in the front, distance to the Chapman–Jouquet plane) on the thermophysical properties of the fuel and oxidant are obtained.

1. It has been established experimentally [1, 2] that the propagation of a detonation process is possible in a tube filled with a gaseous oxidant and on the wall of which is deposited a film of fuel (with a low vapor pressure under the initial conditions). The front of a heterogeneous detonation consists of a shock wave and an extended (not less than two to four tube diameters) combustion zone behind it. A detonation is possible with almost infinitely high ratios of the weight of the fuel on the walls of the tube to the weight of the gaseous oxidant in the volume.

In [3–5], devoted to theoretical consideration of the process, the instantaneous mixing of the fuel entering the volume of the tube with the oxidant was assumed. In [3] the rate of heat release was determined by the kinetics of the chemical reaction of oxidation, and it was concluded that the propagation of the process is impossible without a sufficiently intense supply of fuel. In [4, 5] it was assumed that the fuel entering the volume of the tube from the walls burns instantly, and the Chapman–Jouquet plane was identified with the plane of total vaporization of the fuel film. The velocity defect in the propagation of the process in comparison with the velocity of homogeneous detonation of the same composition was determined by the heat losses and losses to friction. The authors of [4, 5] confined themselves to an examination of processes for "thin" fuel layers, i.e., for ratios of the weight of fuel on the walls of the tube to the weight of gaseous oxidant in the volume close to the stoichiometric ratio.

A schematic picture of the detonation front obtained on the basis of experimental data [5–7] is shown in Fig. 1. Here 0 is the shock front, \( L_1 \) is the distance from the shock front to the joining of the boundary layers, and \( L_2 \) is the distance from the shock front to the plane of completion of heat release.

The following simplifying assumptions are made in the derivation of the equations:

1. The gas density \( \rho \), pressure \( p \), temperature \( T \), and stream velocity relative to the shock front \( u \) are assumed to be constant over a cross section of the tube.

2. The specific gas constant \( R \) and specific heat capacity \( c_p \) of the gas are assumed to be constant over a cross section of the tube.

3. The ratio of heat capacities \( \gamma \) is assumed to be constant over a cross section of the tube, independent of the distance to the shock front, and equal to the value of \( \gamma \) at the Chapman–Jouquet plane for homogeneous gas detonation of a stoichiometric mixture of fuel with oxidant [8].
\[ \gamma = 1.17 \]  

(4) The rate of the chemical reaction is assumed to be infinitely high; the heat release is determined by the processes of turbulent diffusion of the fuel vaporizing into the volume of the tube; the fuel concentration is assumed to be variable over a cross section of the tube.

(5) The heat release \( H_0 \) per gram of oxidant due to the chemical reaction is assumed to be independent of the composition of the products and to equal the corresponding value for homogeneous detonation of a mixture of stoichiometric composition \[ H_0 = D_n \sqrt[2]{2 (\gamma^2 - 1)} c_0' \]  

where \( D_n \) and \( c_0' \) are the detonation velocity and weight concentration of oxidant in a mixture of stoichiometric composition.

(6) The value of the blowing parameter \( B \) [described below by Eqs. (2.1) and (2.2)] is assumed to be independent of the distance to the shock front.

In the coordinate system connected with the shock front the stationary equations of continuity, momentum, and energy with allowance for friction, heat losses, mass supply, and heat release due to the chemical reaction have the form

\[
S_{\rho u} = S_{\rho u} D + \int_0^x m dx
\]

\[
S(\rho u^2 + p) = S(\rho_1 D^2 + p_1) + \int_0^x D_1 f dx + \int_0^x f dx
\]

\[
S_{\rho u} \left( \frac{u^2}{2} + c_p T \right) = S_{\rho u} D \left( \frac{D_1}{2} + c_p T_1 \right) + \left( \frac{D_1}{2} + c_p T_1 \right) \int_0^x m dx + S_{\rho u} D H_0 W + \int_0^x U_1 \frac{q_w dx}{S_{\rho u} D_1}
\]

where \( D \) is the velocity of propagation of the shock front, \( S \) and \( I \) are the cross-sectional area and perimeter of the tube, \( x \) is the distance from the shock front, \( m \) is the rate of mass supply of fuel from a unit of surface, \( \tau_w \) is the shearing stress of friction, \( q_w \) is the heat flux to a unit of surface of the fuel film, \( W \) is the fraction of oxygen which has undergone the reaction, and the index 1 pertains to the initial state of the oxidant.

The equation of state and the expression for the square of the speed of sound have the form

\[ p = \rho R T, \quad a^2 = \gamma RT \]  

Let us change to dimensionless coordinates and to the parameters

\[ \pi = p / p_1, \quad \sigma = \rho_1 / \rho, \quad M = u / a, \quad M_1 = D / a_1 \]

\[ \eta_1 = \int_0^x \frac{m}{S_\rho D_1} dx, \quad \eta_2 = \int_0^x \tau_w / S_\rho D_1 dx, \quad \eta_3 = \int_0^x \frac{q_w}{S_\rho D_1} dx \]

where \( \pi \) is the dimensionless pressure, \( \sigma \) is the dimensionless density, \( M \) is the Mach number of the stream relative to the shock front, \( M_1 \) is the Mach number of the shock front, \( \eta_1 \) is the dimensionless blowing of the flue — the ratio of the mass flow of the vaporizing fuel to the flow of oxidant at the cross section \( x \) of the tube, \( \eta_2 \) is the dimensionless friction, and \( \eta_3 \) is the dimensionless heat losses. By eliminating \( \pi \) and \( \sigma \) from the equations of motion in dimensionless form we obtain an equation of fourth power relative to \( M_1 \):

\[ \frac{\gamma M_1^2 [M_1^2 + 1/(\gamma - 1) + 1 - \frac{1}{2}]}{[(\gamma M_1^2 + 1)^3]^{1/2}} = \frac{1}{2} + \frac{1 - Z}{3 (\gamma^2 - 1)} \]

\[ Z = 1 - 2 (\gamma^2 - 1) \left\{(1 + \eta_1) \left[ \left( \frac{D_1}{D_1} \right)^{\frac{w}{2(\gamma^2 - 1)}} c_0' + \frac{1}{2} (1 + \eta_0) + (\eta_2 - \eta_3) \right] [1 + \eta_1 + \eta_3]^2 - \frac{1}{2} \right\} \]

In the expression for \( Z \) we neglect values on the order of \( (M_1)^{-2} \) in comparison with unity.

By subtracting \( 1/2 \) from both sides of (1.6) we obtain a quadratic equation relative to \( (\gamma M_1^2 + 1) \). Hence,

\[ M_{\pm} = \left( \frac{\gamma + 1}{\gamma} \right)^{1/2} \left[ 1 \pm \sqrt{\frac{1}{2}} \right] \]

The plus and minus signs correspond to the two branches of the solution of Eq. (1.6).