EQUILIBRIUM OF A CRACK IN A POROUS MEDIUM
WITH INJECTION OF THE FILTERING LIQUID

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In connection with the exploitation of petroleum deposits, the article discusses the equilibrium of a porous medium with a crack under conditions of plane deformation, with the steady-state filtration of a liquid injected into the porous medium through a crack. It is assumed that the crack, which has initial zero dimensions, can become wider and longer with a rise in the pressure. The displacement of the sides of the crack is determined on the basis of the theory of elasticity, taking account of the deformation properties of a saturated porous medium. The stress and the displacement are expressed in terms of two analytical Muskhelishvili functions and the complex filtration potential. A change in the volume of the porous medium leads to a discontinuity of the displacements at the feed contour, and to distortion in the filtration region. For a circular stratum, the dimensions of the crack and the mass flow rate of the liquid are determined in the first approximation. The region of values of the pressure in which there exists a stable equilibrium state of the open crack and a steady-state flow of the liquid is found.

1. With the investigation of filtration in a porous medium with a crack, in the case of a moderate pressure of the liquid the deformation of the crack is generally neglected [1, 2]. In [2] the deformation was partially taken into consideration with unsteady-state filtration, when the width of the crack is small compared to its initial value.

With an increase in the pressure of the saturating liquid, the dimensions of the crack can increase considerably. Such conditions exist with the hydraulic fracture of an oil-bearing stratum, and in the case of the flooding of a stratum at a pressure greater than the well pressure [3, 4].

The dependence between the deformations and the stresses in a porous medium can be represented in the form [5]

\[ \varepsilon_{ij} = \frac{1}{2\mu} \left( \frac{2}{1 + \nu} \delta_{ij} - \nu \partial_{ij} \right) + \beta p \delta_{ij} \]

(1.1)

where \( \mu = E/2(1 + \nu) \), \( E, \nu \) are the shear modulus, the elastic modulus, and the Poisson coefficient of the skeleton of the porous medium (the dry rock); \( \beta = \beta_2 - \beta_1 \); \( \beta_1 \) is the coefficient of linear compressibility of the solid phase (the grains making up the porous medium); \( \beta_2 = (1 - 2\nu)/E \) is the coefficient of linear compressibility of the pressure of the porous medium;

The coefficient \( \beta \) depends on the degree of cementing of the rock \( \varepsilon = \beta_1/\beta_2 \) and determines the change in the volume of the porous medium, as a result of a change in the pressure of the liquid, with a constant tensor of the total stresses \( \sigma_{ij} \).

In the limiting case of ideally cemented rocks \( \varepsilon \to 1 \), we have \( \beta \to 0 \). The deformation of such rocks is determined only by the total stresses. We note that, with \( \beta \to 0 \), the porosity \( m \to 0 \).

In another limiting case of soft soils, when \( \varepsilon \to 0 \), the deformation is completely determined by the effective stresses \( \sigma_{ij}^e = \sigma_{ij} - p \delta_{ij} \).

Using (1.1), the stresses and deformations in a porous medium under conditions of plane deformation of the stratum and plane steady-state flow of the liquid can be expressed in terms of two analytical functions \( \varphi(z) \), \( \psi(z) \) and the complex filtration potential \( w(z) \); the rectangular coordinates \( x, y \) are introduced in the plane of the deformation.

\[
\begin{align*}
\sigma_x + \sigma_y &= 4 \text{Re } \varphi' + \sigma_z + 2t \tau_{xy} = 2(\Re \varphi'' + \psi') \\
2\mu \left( (1 + \psi') + l \varphi' \right) &= (3 - 4v) \varphi - z \varphi' - \psi + 2\mu (1 + v) \beta \chi
\end{align*}
\]

(1.2)

where \( k \) is the permeability of the porous medium; \( \eta \) is the viscosity of the liquid.

Formulas (1.2) can be obtained by a method used with steady-state thermal action [6].

2. Let us consider a homogeneous porous stratum with a circular feed contour, at whose center there is a borehole and a symmetrical vertical crack, directed along the \( x \) axis, passing through it. We shall assume that the role of the borehole reduces to a linear source, from which the liquid enters the crack. At the feed contour, the pressure of the liquid is equal to \( p_k \). Outside the feed contour, the pressure is constant or there is no liquid.

A change in the volume of the porous medium in the filtration region leads to a discontinuity in the displacements at the feed contour. Therefore, the problem under consideration reduces to a problem in the theory of elasticity for tightly constituted bodies.

\[
\begin{align*}
\varphi_1 + \left( \frac{\omega}{\omega'} \right) \psi_1 + \psi_1 &= f, \quad \text{Re } w = \frac{k}{\eta} p_k, \quad \zeta \in \gamma_0 \\
\varphi_1 + \frac{\omega}{\omega'} \psi_1 + \psi_1 &= \varphi_2 + \frac{\omega}{\omega'} \psi_2 + \psi_2, \quad \text{Re } w = \frac{k}{\eta} p_k, \quad \zeta \in \gamma_k \\
\left( 3 - 4v \right) \varphi_1 - \frac{\omega}{\omega'} \psi_1 - \psi_1 + 2\mu (1 + v) \beta P &= \left( 3 - 4v \right) \varphi_2 - \frac{\omega}{\omega'} \psi_2 - \psi_2, \quad \zeta \in \gamma_0 \\
P(\zeta) &= \left[ \frac{\eta}{k} \omega (\zeta) - p_k \right] \omega'(\zeta) d\zeta
\end{align*}
\]

(2.1)

The transform \( z = \omega(\xi) \) brings the filtration region to a circular concentric annulus in the plane \( \xi = \rho e^{i\theta} \). The annulus is bounded by the circles \( \gamma_0 \) and \( \gamma_k \), having the radii \( 1 \) and \( P_k \).

Assuming that the crack is small compared with the dimensions of the stratum, we can take \( \omega = \sqrt{2} l (1 + \frac{1}{\xi}) \), where \( l \) is the length of the crack. Then the circle with the radius \( \rho_0 = 2R_k / l \), where \( R_k \) is the radius of the stratum, corresponds to an elliptical contour in the plane of \( z \), close to the feed contour. The deviation of the elliptical contour from the circular contour of the stratum with \( l < 0.2R_k \) does not exceed 0.01 \( R_k \).

For the principal vector of the forces applied to the contour of the crack we have

\[
f = -\frac{1}{2} \int p(\sigma) \left( \sigma - \frac{1}{\sigma} \right) dz, \quad \sigma = e^{i\theta}
\]

(2.4)

where \( p(\sigma) \) is the pressure at the contour of the crack. The tangential stresses due to viscous friction have only a negligible effect on the deformation of the crack.

The volumetric changes in the stratum brought about by injection of the liquid correspond to the distortion. As a result of the symmetry of the problem with respect to the \( x \) and \( y \) axes, there is only a rotational component of the distortion. Therefore,

\[
\begin{align*}
\varphi_1 &= a \frac{Q}{\eta h} \left( \frac{\xi}{\xi} + \frac{1}{\xi} \right) \ln \xi + \varphi_{10}, \quad \psi_1 = \psi_{10}, \quad a = \frac{1 + \sqrt{v}}{1 - \sqrt{v}} \beta
\end{align*}
\]

(2.5)

where \( Q \) is the mass flow rate of the liquid; \( h \) is the thickness of the stratum; \( \varphi_{10} \) and \( \psi_{10} \) are holomorphic between \( \gamma_0 \) and \( \gamma_k \).

The functions \( \varphi_2 \) and \( \psi_2 \) have the form