We have studied the stress–strain state of a transversally isotropic, in a thermal respect, half-space cooled by an opposing-reverse coolant flow. We have analyzed the effect of coolant heating on temperature, deflection, and stresses arising at the surfaces heated by an axisymmetric heat flux.

A cooling system of the mirrors of laser technological units is intended for reducing strains and stress on a heated surface and represents the capillary network with a developed heat-transfer surface. Figure 1 illustrates a capillary cooling system with an opposing-reverse coolant flow. A coolant is fed through one capillary system, arranged perpendicularly to the heated surface, and removed through another. The characteristic hydraulic diameter and thickness of the walls between two neighboring capillaries constitute tenths of a millimeter, and their number in the heated zone amounts to several thousand. Therefore, a classic approach to solving the heat conduction and thermoelasticity problem with a statement of boundary conditions at the capillary walls necessitates excessive computational resources.

Recently such a cooling system has been replaced in modeling by a porous medium in which the coolant flows. Thermophysical characteristics are specified with the help of presented thermal conductivity and heat-transfer coefficients, as well as elasticity moduli determined experimentally. Among the literature cited, such an approach is employed in [1-3] for describing heat exchange in the cooling systems of the mirrors of laser technological units. Their authors assumed a coolant temperature invariable across thickness. However, it is difficult to practically provide the coolant flow rates sufficient to neglect the effect of the coolant heating on the mirror characteristics. The current work is one of the first to have studied the stress–strain state of the capillary cooling systems with a view of the influence of the coolant heating. The regions of varying filtration rate and volume heat-transfer coefficients, where this influence is inessential, have been identified.

The cooling system is regarded as a transversally isotropic, in thermal respect, porous medium with an isotropy plane normal to the capillary direction. Let $T_1$ and $T_2$ be temperatures of the fed and removed coolant. A temperature drop $\Delta T$ across a framework wall is of the order

$$\Delta T \sim \frac{T_2 - T_1}{\frac{\alpha \delta}{\lambda}},$$

where $\alpha$ is the characteristic surface coefficient of heat transfer at the capillary wall, $\delta$ is the wall thickness, and $\lambda$ is the material thermal conductivity.

In the considered cooling systems $\alpha \delta \lambda \approx 0.01$, therefore, it is possible to neglect the temperature drop across the capillary walls and define the framework temperature by a mean temperature $T$. The heat-transfer coefficients and the filtration rate are assumed to be constant throughout the cooling system, and the heat flux is taken to be axisymmetric.

The heat-conduction and heat-transfer problem is formulated as follows. In a half-space $z \geq 0$, three functions $T(r, z)$, $T_1(r, z)$, and $T_2(r, z)$ need to be determined, which satisfy the equations

$$\frac{\partial^2 T}{\partial z^2} + \frac{\lambda_r}{\lambda_z} \frac{\partial^2 T}{\partial r^2} = \alpha_1 (T - T_1) + \alpha_2 (T - T_2),$$

$$\frac{\partial T_1}{\partial z} = \alpha_1 k (T - T_1),$$

$$\frac{\partial T_2}{\partial z} = -\alpha_2 k (T - T_2).$$

On the surface $z = 0$, a heat flux is imposed and the conditions of temperature conjugation for the supplied and removed coolants must be fulfilled

$$-1 \frac{\partial T}{\partial z} = q(r), \quad T_1(r, 0) = T_2(r, 0).$$

The functions $T$, $T_1$, and $T_2$ tend to zero at infinity. The Hankel transform with respect to $r$ yields a system of ordinary differential equations for the transforms of unknown functions $T^*$, $T_1^*$, and $T_2^*$:

$$\frac{d^2}{dz^2} T^* = \alpha_1 (T^* - T_1^*) + \alpha_2 (T^* - T_2^*) + \xi_1 T^*, \quad \frac{d}{dz} T_1^* = \alpha_1 k (T_1^* - T^*), \quad \frac{d}{dz} T_2^* = -\alpha_2 k (T_2^* - T^*). \tag{2}$$

A characteristic equation for this system is of the fourth order:

$$\omega^4 - \omega^2 k (\alpha_1 - \alpha_2) - \omega^2 (\alpha_1 \alpha_2 k^2 + \alpha_1 + \alpha_2 + \xi_1^2) + \omega k (\alpha_1 - \alpha_2) \xi_1^2 + \xi_1^2 k \alpha_1 \alpha_2 = 0. \tag{3}$$

Let the free term in Eq. (3) be positive, then the entire left side is positive for $\omega \to \pm \infty$ and $\omega = 0$. When $\omega = \pm \xi_1$, the left side takes negative values $-\xi_1^2 (\alpha_1 + \alpha_2)$. Therefore, Eq. (3) has four real roots, of which two are positive ($\omega_1$ and $\omega_2$) and two negative ($\omega_3$ and $\omega_4$). Since the intervals of sign reversal are known, the roots are easy to determine numerically.

In some cases, we succeed in writing explicit expressions for the roots.

1. **Thermal insulation of offtakes ($\alpha_2 = 0$):**

$$\omega_1 = \cos \frac{\varphi}{3} \left[ \frac{2}{\sqrt{3}} \sqrt{\frac{k^2 \alpha_1^2}{3} + \frac{\alpha_1 + \xi_1^2}{3}} \right] + \frac{k \alpha_1}{3}, \quad \omega_2, 3 = \cos \left( \frac{\varphi}{3} + \frac{2\pi}{3} \right) \left[ \frac{2}{\sqrt{3}} \sqrt{\frac{k^2 \alpha_1^2}{3} + \frac{\alpha_1 + \xi_1^2}{3}} \right] + \frac{k \alpha_1}{3}, \quad \omega_4 = 0, \tag{4}$$

where

$$\cos \varphi = \frac{V^{3/2}}{k \alpha_1} \left[ \left( \frac{k \alpha_1}{3} \right)^2 + \frac{\alpha_1 - \xi_1^2}{2} \right], \quad \frac{\alpha_1}{3} + \frac{\xi_1^2}{2} + \alpha_1 \right]^2.$$