A mathematical model of hydrodynamics is proposed for filling closed volumes with allowance for air capture by a liquid jet. Adequacy of the model is established by comparison between the calculated and experimental data obtained by means of physical modeling.

It has been noted on physical models that when filling a confined volume by a liquid from above, its hydrodynamics are essentially exposed to air captured by a jet [1, 2]. In this case depending on the amount of air the flow pattern may vary qualitatively.

The data of physical modeling [3] testify the fact that under the real conditions of filling steel-pouring ladles and moulds from above, the amount of air captured by a liquid metal jet is rather significant and its volume may reach the volume of the entering metal. However, in the available mathematical models of hydrodynamic processes, occurring when filling ladles and moulds [4-6], this fact is not taken into account. This situation is, probably, associated with purely technical difficulties in mathematical simulation of a two-phase gas-liquid medium [7].

In the present work we suggest a rather simple mathematical model of hydrodynamics for filling closed volumes with regard to air capture by a metal jet. The adequacy of the proposed model is established by comparison between the calculated and experimental data obtained through physical modeling [8].

The mathematical model for hydrodynamic processes involving a gaseous phase has been proposed earlier in [9] for the case of flow-through of a ladle by an inert gas. In this model two fields of velocities were taken into account: liquid velocities $V_1$ and averaged velocities of the gaseous phase (gas bubbles $V_2$). The two-phase interaction force was determined by their relative velocity $V_{21} = V_2 - V_1$:

$$F_{21} = N_\mu r_n^2 c\mu \frac{\rho_0 V_{21}^2}{2} \frac{V_{21}}{V_{21}},$$

where $\rho_0$ is the liquid density; $C_\mu$ is the hydrodynamic resistance coefficient, in this case the velocity $V_{21}$ is vertically directed upward and its magnitude is expressed by the semiempirical formula:

$$V_{21} = \sqrt{\frac{\sigma}{r_n (\rho_0 - \rho)} + g r_n \left(1 - \frac{\rho}{\rho_0}\right) (1 - \alpha)},$$

where $\alpha = 4/3 \pi r_n ^3 N$ is the gas content coefficient; $\sigma$ is the surface tension coefficient of the liquid; $\rho = \rho_0 (1 - \alpha)$ is the gas-liquid mixture density; $g = 9.8$ m/sec$^2$.

The presence of two empirical parameters $C_\mu$ and $\tilde{v}$ (which, in addition, depend on the flow pattern) significantly degrades the usefulness of the two-velocity model in practical computations because there are not any reliable numerical data for $C_\mu$ and $\tilde{v}$. In this case, following the authors of [9], we may use numerical experimentations and fit parameters $C_\mu$ and $\tilde{v}$ in such a way to finally obtain correct values of the experimentally verified quantities, for example, of velocity fields for the physical model. However, some other parameters, for instance, the turbulence parameters (on which $C_\mu$ and $\tilde{v}$ are also dependent)
Fig. 1. Calculation results on filling the vessel with account of air injection by the jet at $V_0 = 1$ m/sec and $c_0 = 0.1$; a) field of velocities with indication of their values, cm/sec; b) curves of vertical velocities, $V_z$; c) lines of air isoconcentration, %.

exert an effect on the velocity fields, therefore it is practically impossible to assess the contribution of one or the other parameter to the final pattern. Moreover, because of the problem nonlinearity complicated by the nonlinear dependence of $C_p$ and $\dot{\nu}$ on the motion parameters, it is rather difficult to extend these results to other regions that can not be immediately verified experimentally.

That is why in the present work we formulate the mathematical model which eliminates the need to the maximum possible extent to use the difficult-to-determine parameters and which takes into account only the main physical factors affecting the flow pattern of the two-phase gas–liquid medium in the conditions of our problem.

Contrary to the authors of [9], we do not describe transfer of the gaseous and liquid phases separately with two different velocity fields for each of phases, but in the combined one-velocity approach via the supposition on the continuity of the single gas–liquid medium, being density-stratified by the viscous incompressible liquid. In this case there is no longer a need to make any suppositions concerning the shape and sizes of the gas bubbles.

The motion equations for such medium take on the form [10]

$$\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla) \mathbf{V} = -\frac{1}{\rho} \nabla p + \frac{\mu}{\rho} \Delta \mathbf{V} + \mathbf{g};$$

(1)

$$\nabla \cdot \mathbf{V} = 0;$$

(2)