Flow and heat transfer at mixed convection in the vertical channel connecting a cryogenic vessel and a room temperature zone are considered. The two-dimensional problem of conjugated heat transfer in the metal wall of the channel and in its cavity is solved by the finite difference method. The calculated values of the heat flux into the cold zone and of the temperature of the hot pipe end at different channel wall thicknesses, lengths, diameters, helium flowrates, as well as at different constants of the interaction of heat with the environment are given.

The outlet pipe of the cryostatting system for the helium temperature level can initiate considerable heat fluxes into the cold cryostat zone under certain conditions. This problem is also significant for a gas-cooled cryogenic current lead, whose element can be considered as a pipe, with its wall being electrically heated. Strictly speaking, this study is concerned with a pipe since the shape of gas channels in real designs of cooled current leads may be sufficiently complex.

In the available publications dealing with current lead calculations, metal-to-coolant heat transfer coefficients are assigned a priori, laying stress on optimization of geometrical sizes under sufficiently good or ideal heat transfer. Theoretically, the problem of providing such heat transfer conditions is not considered.

In the majority of cases, the experimentally measured heat fluxes into the cold zone for current leads greatly exceed the design ones [1]. Such a difference can be caused by the fact that the contribution of free convection imposed on forced flow has not been taken into account. Meanwhile, this mechanism is insufficiently studied in publications on cooling of heat bridges and current leads.

Usually, flow from cryogenic vessels occurs at low Reynolds numbers, and temperature drops both in the longitudinal direction (hot end — cold end) and in the cross direction (flow axis — heated wall) attain a hundred degrees. These circumstances and the specific features of helium thermophysical properties result in conditions for mixed convection to occur in outlet pipes.

The previous publication [2] is concerned with calculation results for a nonelectrically heated pipe. The present work considers a wall-heated pipe.

The mathematical statement of the problem includes the Navier—Stokes continuity equation and the heat transfer equation in a cylindrical coordinate system

\[ \frac{\partial}{\partial x} (r p u) + \frac{\partial}{\partial r} (r p v) = 0, \]  
\[ \frac{\partial}{\partial x} (r p u^2) + \frac{\partial}{\partial r} (r p u v) = -r \frac{\partial p}{\partial x} + \frac{\partial}{\partial x} \left( r \mu \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial r} \left( r \mu \frac{\partial u}{\partial r} \right) - \rho g, \]  
\[ \frac{\partial}{\partial x} (r p v u) + \frac{\partial}{\partial r} (r p v^2) = -r \frac{\partial p}{\partial r} + \frac{\partial}{\partial x} \left( r \mu \frac{\partial v}{\partial x} \right) + \frac{\partial}{\partial r} \left( r \mu \frac{\partial v}{\partial r} \right) + \frac{2 \mu v}{r}, \]
\[
\frac{\partial}{\partial x} (\rho \nu c_p T) + \frac{\partial}{\partial r} (\rho \nu c_p T) = \frac{\partial}{\partial r} \left( \rho h \frac{\partial T}{\partial r} \right) + \\
+ \frac{\partial}{\partial x} \left( \rho h \frac{\partial T}{\partial x} \right) + \mu r \left( \frac{\partial u}{\partial r} \right)^2 + \rho u \frac{\partial P}{\partial x}
\]

subject to the boundary conditions: on the cold pipe end at \(x = 0\)

\[
u = u_0, \quad v = 0, \quad T = T_0.
\]

on the hot pipe end at \(x = L\) and within \(a < r < b\)

\[-\lambda \frac{dT}{dx} = \alpha_1 (T_1 - T_0),
\]

on the hot pipe end at \(x = L\) and within \(0 < r < a\)

\[\frac{\partial u}{\partial x} = 0, \quad \frac{\partial T}{\partial x} = 0, \quad v = 0,
\]

on the inner pipe surface \(r = a\)

\[\kappa \left. \frac{\partial T}{\partial r} \right|_{r=a^-} = \kappa \left. \frac{\partial T}{\partial r} \right|_{r=a^+}, \quad u = 0, \quad v = 0,
\]

on the outer pipe surface \(r = b\)

\[-\lambda \frac{dT}{dr} = \alpha_2 (T_b - T_0),
\]

on the symmetry axis \(r = 0\)

\[\frac{\partial u}{\partial r} = 0, \quad \frac{\partial T}{\partial r} = 0, \quad v = 0.
\]

Boundary conditions (6) and (9) are generalizations of the more frequently reported conditions for ideal heat transfer on the hot end \(T_1 = T_e\) and of the conditions for no-background heat input in the cryostat \(q_3 = 0\).

The problem of the type of boundary condition at \(x = x_1\) is ambiguous. In a correctly designed current lead the condition \(T_1 = T_e\) is satisfied, however, rather not due to a great coefficient for the interaction between heat and surrounding medium \(\alpha_1\), but due to a successfully chosen relation between the longitudinal size of the current lead and the cross section of a current-conducting metal wall. On the other hand, in practice the condition \(T_1 = T_e\) is more frequently violated since, first, the service conditions (i.e., current and flowrate) can differ from the design ones and, second, the empirical knowledge about the heat transfer coefficient included into the design of the current lead in terms of this or that optimization may not conform to reality.

The system of equations (1)-(4) with the boundary conditions was solved by Patankar’s method [3] as described in [2]. Sebesi and Smith’s model [4], being most complete among the algebraic ones and well checked for tube flow, was used to calculate effective values of the viscosity coefficient and thermal conductivity.

Helium thermophysical properties were assigned allowing for tabular data [5], and we considered copper M1 as the pipe wall material, whose approximate physical properties were given in [1].

The present study is mainly aimed at revealing the specific features of flow and heat transfer that follow from the two-dimensional nature of processes and are bound up with vortex formation, which cannot be taken into account in the one-