PULLING OF GLASS MICROCAPILLARIES: THEORY AND EXPERIMENT

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An approximate analytical solution is obtained which predicts a microcapillary form during pulling which is close to that observed in experiments.

Microcapillaries (hollow fibers) find wide application in opto- and radioelectronics, artificial kidney apparatuses, etc. In [1], an analytical theoretical model is proposed of pulling of glass microcapillaries from a tubular workpiece which is heated as it passes through a furnace. (Numerical solutions are also known of the problems on pulling of hollow fibers [2-4]; in the last work a dynamic problem is solved without consideration of heat transfer and viscosity variation with temperature.) In the present work the predictions made by the model [1] are compared with experimental data. As a result, it is possible to determine the configuration of a fiber narrowing zone in the pulling process and to evaluate its dimensions.

First of all we briefly dwell on the pulling model and its theoretical description. Figure 1 shows how a glass tube (a workpiece) is brought into the furnace, heated (as a result, glass becomes soft and starts flowing), and is stretched, thus becoming thinner under the action of the force created by the receiving device. Upon leaving the furnace, the glass is cooled, gradually becoming a solid, and the process of fiber extension practically ceases.

Assuming the workpiece and microcapillary walls to be sufficiently thin, we use quasi-two-dimensional (becoming quasi-one-dimensional in virtue of axial symmetry) equations of dynamics of thin films to describe a steady-state glass flow in the molded fiber (see [1, 5-7]). In the given case they are reduced to the form

\[ R \rho V \frac{dV}{dx} = Q; \]  

\[ \rho Q \frac{dV}{dx} = \frac{d}{dx} (\Sigma_{\tau\tau} R h) - \Sigma_{\theta\theta} h \frac{dR}{dx} + \rho g h R; \]  

\[ \rho Q V \lambda k = \Sigma_{\tau\tau} R h \lambda k - \Sigma_{\theta\theta} h + 2a (R \lambda k - 1) - \rho g R h \frac{dR}{dx}; \]  

\[ \rho c Q \frac{dT}{dx} = - (q_{v1} + q_{v2}) \lambda R; \]  

\[ \Sigma_{\tau\tau} = 2\mu \left( \frac{2}{\lambda} \frac{dV}{dx} + \frac{V}{\lambda R} \frac{dR}{dx} \right); \]  

\[ \Sigma_{\theta\theta} = 2\mu \left( \frac{1}{\lambda} \frac{dV}{dx} + \frac{2V}{\lambda R} \frac{dR}{dx} \right); \]
The first equation in (1)-(9) is the continuity equation, the second and the third are the projections of the equation of momentum onto the tangent and normal to the generatrix (liquid glass — viscous Newtonian fluid), and the fourth is the heat transfer equation (heat transfer by conduction along a fiber is negligible). As a rule, \( q_{p2} = 0 \) may be taken. The pressure in the fiber cavity is assumed to be equal to the external pressure.

In equations of motion (2) and (3), the terms of inertia forces, weight, and surface tension may be neglected. Besides, for thin, gradually becoming thinner fibers \( \lambda = 1, kR \ll 1 \); as a result, the third equation gives \( \Sigma_{e\theta} = 0 \). With regard for this, an approximate analytical solution of the system (1)-(9) is constructed by the Laplace method, as it has been done earlier for solid fibers [8], on the assumption that the activation energy in the Arrhenius law for viscosity is high. In the initial fiber section, its radius, wall thickness, velocity, and temperature \( R_0, h_0, V_0, T_0 \) are prescribed, while in the terminal section (on the receiving device) a reception rate \( V_1 \) is given. As a result, the following approximate analytical solution is obtained in the parametric form:

for the heating zone \( (0 \leq x \leq l, T_0 \leq T \leq T_p) \)

\[
\bar{R} = \bar{n} = 1 - (1 - m) \exp \left[ \theta \left( \bar{T} - 1 \right) \right];
\]

\[
x = -\frac{T_p \rho_c Q}{q_1 R_0} \left\{ \bar{T} - \bar{T}_0 - \theta^{-1} \ln \left[ 1 - (1 - m) \exp \left[ \theta \left( \bar{T} - 1 \right) \right] \right] \right\};
\]

for the cooling zone \( (l \leq x \leq L, T_p \geq T \geq T_1) \)

\[
\bar{R} = \bar{n} = m - (E^{-1/2} - m) \left\{ \exp \left[ \theta \left( \bar{T} - 1 \right) \right] - 1 \right\};
\]

\[
x = l - \frac{T_p \rho_c Q}{q_1 R_0} \left( \frac{q_2}{q_1} \right)^{-1} E^{1/2} \times
\]

\[
\times \left\{ \bar{T} - 1 - \theta^{-1} \ln \left[ m^{-1} E^{-1/2} - (m^{-1} E^{-1/2} - 1) \exp \left[ \theta \left( \bar{T} - 1 \right) \right] \right] \right\};
\]

In both regions

\[
\bar{V} = \bar{R}^{1/2};
\]

\[
m = \frac{1 - E^{-1/2} \left( \frac{q_2}{q_1} \right)}{1 - \frac{q_2}{q_1}},
\]

where \( \bar{R} = R(x)/R_0; \bar{n} = h(x)/h_0; \bar{V} = V(x)/V_0; \bar{T} = T(x)/T_p; \bar{T}_0 = T_0/T_p; \bar{T}_1 = T_1/T_p; \theta = u/(G T_p); E = V_1/V_0 \) is the pulling ratio; \( q_1 = q_{p1} < 0 \) (heating) at \( 0 \leq x \leq l; q_2 = q_{p2} > 0 \) (cooling) at \( l \leq x \leq L \).