An approximate model of melting of semitransparent media with formation of an isothermal transition zone is constructed which makes it possible to establish possible regimes of melting and to analytically calculate the dynamics of the process under the action of pulse radiation flux.

By semitransparent (or, in other words, partially transparent) materials are meant substances which, in certain regions of the spectrum, transmit thermal radiation incident on them for a considerable distance into the depth of the specimen [1]. Practically all dielectrics in various structural states are among these materials, including ice and snow, which are widespread in nature. Under radiation flux incident on the outside of the specimen of some medium, the latter can attain the melting temperature and then undergo a phase transition. If the medium is nontransparent in relation to radiation (metals are an example practically over the entire range of the thermal radiation spectrum), then its melting starts from the surface: the melting front divides the melt and the solid phase. A mathematical description of this process is given by different variants of the classical Stefan problem, to which much research is devoted (see, for example, [2-6]).

To describe a phase transition in semitransparent media under thermal radiation, including the intrinsic radiation of the substance, use was also made of the classical Stefan problem [1, 7, 8]. The employed methods for specifying the boundary condition on a moving boundary are surveyed in [9]. However, as a theoretical analysis [10] and numerical calculations [8] show, under certain conditions paradoxical situations occur, which invoke no explanation within the framework of the classical Stefan problem — the zones of superheating of the solid phase emerge. The analysis leads to the following physical picture of melting of semitransparent bodies. Having absorbed the radiation penetrating into the body and having attained the melting temperature, the volume element of the partially transparent substance is transformed into a liquid state not instantly but in some time. This time is determined by the intensity of the radiation source and by thermophysical and optical properties of the material. In the semitransparent material layer three zones are formed: a solid which has not attained the melting temperature, an intermediate isothermal zone, formed by a mixture of the melt and the solid phase with different volume content of the melt, and, finally, a zone of the totally melted material. The emergence of the intermediate zone formed in the interaction of radiation with semitransparent media was observed and described in [9]. According to this physical picture a mathematical formulation of relevant problems is also bound to be changed.

Generalized statements of the problems allowing for the presence of the intermediate zone are considered in [9-18] as applied to the problems of melting and crystallization, thawing of frozen grounds, and to geophysical problems.

The present work considers in a one-dimensional statement the approximate model of melting of a semitransparent medium under pulse irradiation. By pulse radiation, according to [19], we will mean radiation emitted for a limited, fairly small time. Gas-discharge pulse lamps, strong-current carbon arcs, power incandescent lamps, and others [19] as well as luminous bodies produced in large-scale accidents and explosions [20] may serve as pulse radiation sources of moderate intensity. The operating time of the mentioned sources is as a rule of the order of $10^{-2}-10^0$ sec, the characteristic time $t_0$ of irradiation rise to the maximum is fractions of a second, and the maximum irradiation $F_0$ is of the order of $10^5-10^8$ W/m$^2$. The model of melting a semitransparent medium discussed below takes account of the pulse character of irradiation and enables us to analytically calculate the dynamics of the process of melting and the limiting thickness of the melted layer.

1. The basis for the approximate model of melting of semitransparent media is formed by the following assumptions. It is believed that in melting there occurs no variation in the density of the material so as not to take into account convective streams arising with different densities; the refractive indexes of the solid and the melt are similar and there is no internal reflection at the solid—melt interfaces. The approximation of a gray substance is taken. It is believed that the intermediate iso-
thermal zone formed by a mixture of the melt and the solid phase has an extinction coefficient, independent of the melt fraction. In connection with the given assumption we will point out that the physical properties of the transient zone formed in melting or crystallization of semitransparent bodies have hardly been investigated to date [9, 11]. The intrinsic radiation of a medium is assumed to be negligible in comparison with the absorbed radiation (approximation of a cold substance). To describe the extinction of the radiation flux in the bulk of the semitransparent substance we employ Bouguer’s law with an effective extinction coefficient independent of the medium temperature. Finally, the pulse character of irradiation with rather high intensity assumes rapid heating of the medium when heat conduction is of negligible importance.

We will consider in more detail the last assumption, which is of substantial importance in constructing the approximate model of melting. Assume that there is given a half-space \( x \geq 0 \), filled with a partially transparent substance with the extinction coefficient \( a_1 \), the reflectivity \( R_1 \), and the initial temperature \( T_0 < T_c \). For \( t \geq 0 \) the half-space is externally irradiated by the pulse radiation flux described by the function \( F(t) = F_0 f(t/\tau_0) \). Using the dimensionless variables \( u = (T - T_0)/\Delta T \), \( \theta = t/\tau_0 \), and \( y = a_1 x \) the mathematical formulation of the problem of heating up the half-space with constant thermophysical characteristics until its elements attain the melting temperature has the form

\[
\frac{\partial u}{\partial \theta} = b \frac{\partial^2 u}{\partial y^2} + d_1 \exp(-y) f(\theta),
\]

\[
y \geq 0, \quad 0 < \theta < \theta_m.
\]

\[
u(y, 0) = 1, \quad y \geq 0,
\]

\[
\frac{\partial u}{\partial y} (0, \theta) = 0, \quad \theta > 0.
\]

Here

\[
b = \frac{\kappa a_1^2\tau_0}{\rho c}, \quad d_1 = \frac{a_1 (1 - R_1) F_0 \tau_0}{\rho c \Delta T},
\]

\( \theta_m \) is the dimensionless time of attaining the melting temperature. The condition (3) characterizes the absence of heat exchange between the half-space and the ambient medium. In many cases the dimensionless coefficient \( b \) is a small quantity. For example, for ice \( \kappa = 2.23 \text{ W/(m·K)} \), \( a_1 = 5 \text{ m}^{-1} \) [21], \( \rho = 917 \text{ kg/m}^3 \), \( c = 2.1 \times 10^3 \text{ J/(kg·K)} \). Assuming \( \tau_0 = 1 \text{ sec} \), we obtain \( b = 2.9 \times 10^{-2} \). In this connection along with problem (1)-(3) we will deal with the following Cauchy problem, formally corresponding to the case \( b = 0 \):

\[
\frac{\partial U}{\partial \theta} = d_1 \exp(-y) f(\theta), \quad \theta \geq 0,
\]

\[
U(y, 0) = 1, \quad y \geq 0.
\]