EFFECT OF GAS COMPRESSIBILITY ON THE STABILITY OF A BOUNDARY LAYER ABOVE A PERMEABLE SURFACE AT SUBSONIC VELOCITIES

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In the present study we investigate the stability of a boundary layer for the condition that the velocity perturbations at the permeable surface are nonzero. The stability for the boundary layer of an incompressible liquid in such a formulation was considered in [1]. For the case of subsonic velocities the effect of compressibility on the flow inside the boundary layer is weak, and in the present article this effect was neglected. The unsteady flow in narrow pores of a permeable covering depends strongly on the compressibility of the gas. Therefore, in the derivation of the relation connecting the pressure oscillations at the permeable surface with the oscillations of the flow through it, the effect of the compressibility was taken into consideration. It is shown that the boundary conditions, and therefore also the stability of the boundary layer at the permeable surface, depend considerably on the Mach number, even for a subsonic exterior flow.

1. The stability of a boundary layer of a compressible liquid at subsonic velocities above an impermeable surface was investigated in [2, 3]. It was shown that the characteristics of the stability over a thermally insulated surface depend weakly on the Mach number. This is explained, on the one hand, by the fact that in the absence of heat exchange the distribution of the mean velocity in the boundary layer of subsonic velocities differs weakly from the velocity distribution for \( M = 0 \), the value of the temperature over the entire layer being approximately constant and equal to the temperature at the outer boundary of the boundary layer [4], and, on the other hand, by the fact that the temperature perturbations in the boundary layer can be neglected [2].

Therefore, in the absence of heat exchange at subsonic velocities the distribution of the perturbation amplitude of the stream function \( \psi = \varphi(y) \exp \left[ i \alpha (x - ct) \right] \) approximately satisfies the Orr-Sommerfeld equation

\[
(U - c) \left( \psi'' - \alpha^2 \psi \right) - U' \psi = \frac{4}{i \alpha \text{Re}} \left( \psi' \psi - 2 \alpha^2 \psi' \psi' + \alpha^2 \psi \right). \tag{1.1}
\]

The usual notation is used here [2, 4]. Equation (1.1) should be solved with four boundary conditions. According to [1], these conditions are

\[
\begin{align*}
\varphi (\infty) &= \varphi' (\infty) = 0, \\
\varphi' (0) &= 0, \quad (U' (0) - i \alpha K) \psi (0) = - \frac{1}{i \alpha \text{Re}} \psi'' (0).
\end{align*} \tag{1.2}
\]

The first two conditions are the conditions of damping of the perturbations at infinity, the third is the condition of nonpassage along the surface (the plate is permeable only in the normal direction). The fourth condition is obtained from the equations of motion and the law of permeability,

\[
v(0) = - K p(0). \tag{1.3}
\]

Here \( v(0) \) and \( p(0) \) are dimensionless perturbations of velocity and of pressure in the boundary layer near the surface of the permeable plate; \( K \) is a coefficient of proportionality, determined below. The pressure...
is referred to the total head at the boundary of the boundary layer $\rho U_e^2$, and the velocity is referred to the velocity at the boundary of the boundary layer $U_e$. To determine $K$ we consider the model proposed in [1]. The pores of the permeable plate have a cylindrical shape and are oriented in the normal direction to the plate surface. Thus, we consider a perforated plate with small diameters of the holes and distances between them, at least in comparison with the thickness of the boundary layer. The dimensions of the holes (pores) are large enough so that we can take the pressure distribution in them to be independent of the radial direction. On the basis of this model, an expression for $K$ was obtained in [1] for an incompressible liquid.

For the case of a compressible gas, in order to determine $K$ we should use the theory of propagation of acoustic waves in long narrow channels. The propagation of acoustic waves is characterized by a propagation constant $\lambda$ and characteristic impedance $Z_0$. By analogy with the electrically conducting line in [5] we obtain values of $\lambda$ and $Z_0$, expressed in terms of acoustic parameters: $Z$, the impedance of an element of pipe, and $Y$, a coefficient characterizing the margin of energy of compression and loss of thermal energy of a tube element due to heat transfer to the walls of the pipe. The acoustic parameters characterize the relation between the bulk velocity and pressure. In the present study it is convenient to consider the relation between the velocity $V_1$ and the pressure averaged over the pipe cross section. For a law of propagation of velocity and pressure averaged over the cross section along a long pipe we evidently can use an analogy with an electrically conducting line having impedance $SZ$ and a second parameter $Y/S$, where $S$ is the cross-sectional area of the pipe. Using $Z$ and $Y$, taken from [6], we can write the dimensionless quantities $Z_1 = Z - 6/\rho U_e$ and $Y_1 = Y/\rho U_e 6/S$ as

$$Z_1 = i\alpha \frac{\lambda_0 (\nu + 1)}{\lambda_0 (\nu + 1)}$$

$$Y_1 = - i\alpha \frac{\lambda_0^2 (\nu + 1)}{\lambda_0^2 (\nu + 1)}$$

As dimensional quantities we use the quantities earlier assumed in the present study. It was taken into account that the frequency $\omega = -\alpha c U_e / \delta$, where $\delta$ is the thickness of the boundary layer. In (1.4) we assume the following notation: $M_0$ is the ratio of the flow velocity at the external boundary of the boundary layer to the acoustic velocity near the surface, $\sigma$ is the Prandtl number, $\alpha$ is the adiabatic exponent, $r_1$ is the ratio of the radius of a hole (pore) to the thickness of the boundary layer, and $I_0$ and $I_2$ are Bessel functions of order zero and order two. We should note that in the present study we consider cases in which the temperature distribution over the layer is approximately constant; therefore, by $M_0$ we shall mean the Mach number of the incoming fluid. By analogy with [5] the propagation constant $\lambda$ and the characteristic impedance $Z_0$ are defined by the equations

$$\lambda = (Z_1 Y_1)^{1/2}, \quad Z_0 = Z_1 / \lambda.$$

On one end of the pipe (pore) let us be given the relation

$$p(-H) = X_1 \cdot v_1(-H).$$

Then by analogy with the results presented in [7] we can obtain

$$\frac{v_1(0)}{\rho(0)} = -\frac{1}{Z_0} \frac{Z_0 - X_1 \cdot \text{th}(\lambda H)}{Z_0 \cdot \text{th}(\lambda H) - X_1}.$$

If the fraction of the surface occupied by the holes is $n$, then the velocity near the surface $v(0) = n v_1(0)$; therefore,

$$K = -\frac{v(0)}{\rho(0)} = \frac{n}{Z_0} \frac{Z_0 - X_1 \cdot \text{th}(\lambda H)}{Z_0 \cdot \text{th}(\lambda H) - X_1}.$$

The value of $X_1$ is determined simply if condition (1.5) is written for the end of the pipe (pore) adjacent to a large volume in which the gas has no average motion (e.g., a chamber of weak suction). Using the equation $v_1(-H) = v(-H)/n$ (n is the porosity, $H$ is the thickness of the permeable covering, and $v(-H)$ are the velocity perturbations near the permeable covering), according to [4], we can obtain

$$X_1 = X/n = (i\alpha - \alpha^2/\text{Re}) (\alpha + \gamma) / \alpha^2, \gamma = -v' - i\alpha c \text{Re} + \alpha^2.$$

2. In certain particular cases we can qualitatively explain the effect of permeability of perturbations through the surface on the stability of the boundary layer.