ESTIMATES OF THE NORMAL VELOCITIES OF PROPAGATION OF LAMINAR AND VERY SMALL-SCALED TURBULENT FLAMES

V. S. Baushev and V. N. Vilyunov

On the most general assumptions (taking account of the Lewis–Semenov number, thermal expansion, variability of thermophysical parameters, etc.), analytical estimates are obtained for the normal velocities of combustion of laminar and turbulent flames. In the case of an Arrhenius dependence of the reaction velocity on the temperature, the combustion velocity is represented by an asymptotic series with respect to the Frank-Kamenetski dimensionless temperature; for turbulent flames, with respect to a parameter of the relative scale of turbulence. The final results over a wide range of change of parameters are compared with a numerical calculation on a computer of the exact equations and with the relations obtained by the method of combined asymptotic expansions.

1. Mathematical Formulation of the Problem. Laminar Flame

When the temperature dependence of the rate of the volume heat release is determined by the Arrhenius law

\[ \Phi = (\rho(T))^\alpha \exp(-E/RT), \] (1.1)

the thermal diffusion mechanism of propagation of a one-dimensional steady flame is described [1] by the system of equations

\[ \frac{dp}{du} = \nu k(u) \phi(u)/p - \omega; \] (1.2)

\[ (1/L) \frac{dv}{du} = 1 - \omega(v - u)/p, \quad 0 < u < 1 \]

and with the boundary conditions

\[ u = 0, \quad p = 0, \quad v = 0; \] (1.3)

\[ u = 1, \quad p = 0; \] (1.4)

\[ \phi(u) = \begin{cases} \exp(-\theta u/(1 - au)), & 0 \leq u \leq \varepsilon \\ 0, & \varepsilon < u \leq 1. \end{cases} \] (1.5)

The "cutoff" equation (1.5) of the heat release (\( \varepsilon \) is the "cutoff" parameter) ensures the existence of an eigenvalue \( \alpha_0 \) of the problem (1.1)-(1.4), which is unique when \( 1 \leq Le \leq \varepsilon \) [1]. The question of uniqueness when \( Le < 1 \) still does not have a solution.

The relations between the dimensionless and dimensional quantities are

\[ u = (T_+ - T)/(T_+ - T_-); \quad p = -(\lambda/\lambda_+) du/dz_0; \quad \xi = x/x_+, \quad k(u) = (\lambda/\lambda_+)(\rho/\rho_+)^{\alpha\xi^2/2}; \]
where \( v \) is the concentration; \( T \) is the temperature; \( w_+ \) is the normal velocity of flame propagation, relative to the reaction products; \( c = \text{const} \) is the specific heat at constant pressure; \( \rho = \rho(T) \) is the density; \( D = D(T) \) is the effective coefficient of diffusion; \( n \) is the reaction order; \( x_+ \) is the spatial scale; \( z(T) \) is the frequency factor; \( \text{Le} \) is the Lewis–Semenov number; \( \tau_+ \) is the characteristic time of chemical reaction; \( E \) is the energy of activation; \( \beta \) is the gas constant. We denote the parameters referring to the initial mixture and to the final reaction products by the subscripts minus and plus, respectively. A similar indexing is used later for denoting the upper and lower bounds of functions and velocities of combustion.

In the special case of power functions \( \lambda \sim T^{m_1}, \rho \sim T^{-n}, z \sim T^{m_2}, 0 \leq m_1 \leq 1, 0 \leq n \leq 3, 0 \leq m_2 \leq 1 \), we have

\[
k(u) = (1 - \sigma) m_1 m_2 = m_1 - n + m_2.
\]

Further consideration is valid also in the case when the temperature dependence \( f(u) \) differs from the Arrhenius dependence, but satisfies the conditions for the existence and uniqueness of the eigenvalue \( \omega_0 \).

The case of the dependence of the solutions of system (1.2) with the conditions (1.3) on \( \omega \) is denoted by the following equations:

\[
p(u) = \tilde{p}(\omega, u); \quad v(u) = \tilde{v}(\omega, u).
\]

The boundary condition (1.4), taking account of Eq. (1.5), is equivalent to the condition

\[
p(\varepsilon) - \omega(1 - \varepsilon) = 0.
\]

Therefore, instead of Eq. (1.4), Eq. (1.7) can be used, and the solutions of system (1.2) can be considered only in the region \( 0 < u < \varepsilon \).

2. Estimates of Combustion Velocities. Laminar Flame

When \( \text{Le} = 1 \), system (1.2) reduces to a single equation:

\[
dp/du = q(u)/\rho - \omega, \quad q(u) = u^m k(u)/u.
\]

We shall assume the "cutoff" parameter to be variable and we shall denote it by \( t, 0 < t < \varepsilon \). The eigenvalue \( \omega_0 = \omega_0(t) \) will be satisfied according to Eq. (1.7) by the equation

\[
\tilde{p}(\omega_0, t) - \omega_0(1 - t) = 0.
\]

Differentiating this equation with respect to \( t \), and taking into account that according to Eq. (1.6)

\[
\partial p/\partial t + \omega_0 = \lim_{u \to \infty} (dp/du + \omega_0) = q(t)/\tilde{p}(\omega_0, t) = q(t)/\omega_0(1 - t),
\]

and denoting \( q(t) = \tilde{q}(\omega_0, t) = \partial \tilde{p}(\omega_0, t)/\partial \omega_0 \), we obtain the differential equation for \( \omega_0 \):

\[
d\omega_0/\partial t = q(t)/[\omega_0(1 - t)[1 - t - \tilde{q}(\omega_0, t)]
\]

with the condition \( \omega_0(0) = 0 \), which follows from Eq. (2.2)

According to the theorem of estimates [2], with increase of \( \omega \) the solution of Eq. (2.1) with the condition \( p(0) = 0 \) is reduced; therefore, \( q < 0 \). Differentiating Eq. (2.1) with respect to \( \omega \), we obtain

\[
dq/\partial t = -(q(u)/\rho^2) q - 1.
\]

When \( u = 0, q = 0 \), and therefore

\[
q(t) = \int_{\delta}^{\infty} \frac{q(u)}{u^2} q du - t > -t.
\]

After substitution in Eq. (2.3) of the upper \( (q_+ = 0) \) and lower \( (q_- = -t) \) functions and after subsequent integration, we find the upper and lower estimate of \( \omega_0 \):

\[
\omega_0^2 = 2 \int_{\delta}^{\infty} \frac{\varphi(u)}{(1 - u)^2} du; \quad \omega_0^2 = 2 \int_{\delta}^{\infty} \varphi(u) \frac{du}{1 - u}.
\]

\[
\omega_0\]