THERMAL STRESSES IN TUBES, PRODUCED FROM A MELT BY THE STEPA NOV METHOD, DURING THEIR COOLING

A. V. Zhdanov and L. P. Nikolaeva

We calculated the temperature fields and the corresponding thermoelastic stresses in tubes, produced from a melt by the Stepanov method, during their cooling. The results of the calculations are presented in the form of surfaces constructed above the longitudinal cross section of the tube. We investigate the maximum values of stresses as a function of the rate of cooling and the behavior of the difference between the temperatures of the ambient inside and outside the tube.

Introduction. A number of publications, e.g., [1-3], present calculations of the temperature fields and corresponding thermal stresses in tubes, obtained from a melt by the Stepanov technique, during the growth of these tubes. In the present work we suggest calculations of thermoelastic stresses in tubes during their cooling, i.e., stresses varying in time. We also follow the tendency of the variation of these stresses as a function of such important parameters as the differences in the temperatures of the media inside and outside the tube. To determine the temperature field \( T(r, z, t) \), varying in time, in a cooling-off crystal, it is necessary to know the initial distribution of the temperature \( T^0(r, z) \). It was found in [1] and is used here.

For clarity we will show the distribution of the normal meridian \( \sigma_m \) and normal circumferential \( \sigma_\phi \) stresses in the form of surfaces constructed above the longitudinal cross section of the tube at different instants of cooling. The calculations obtained permit one to obtain the initial data for optimizing the cooling process, primarily associated with the behavior of stresses in tubes and with the time of this process.

1. Mathematical Model. During the crystallization of a tube of length \( L \), with the inner radius \( R_1 \) and the outer radius \( R_2 \), produced with the speed of pulling \( V_0 \), the temperature field in it \( T^0 \) satisfies the quasistationary equation of heat transfer

\[
\frac{1}{r} \left( \frac{\partial}{\partial r} \left( r \frac{\partial T^0}{\partial r} \right) \right) + \frac{\partial^2 T^0}{\partial z^2} - V_0 \rho_s c_p \frac{\partial T^0}{\partial z} = 0
\]

subject to the following boundary conditions: on the inner and outer surfaces of the tube we prescribe heat exchange with the surrounding medium having the temperatures \( \Theta_1 \) and \( \Theta_2 \) (see Fig. 1):

\[
- k_s \frac{\partial T^0}{\partial r} = h_s (T^0 - \Theta_2) \big|_{r=R_2}, \quad k_s \frac{\partial T^0}{\partial r} = h_s (T^0 - \Theta_1) \big|_{r=R_1},
\]

where

\[
\Theta_1^0 = T_1^0 + \frac{z}{l} (T_2^0 - T_1^0), \quad \Theta_2^0 = T_3^0 + \frac{z}{l} (T_4^0 - T_3^0).
\]

On the lower (the front of crystallization) and upper ends of the tube the following temperatures are prescribed:

\[
T^0(r, 0) = T_m^0, \quad T^0(r, l) = T_c^0, \quad R_1 \leq r \leq R_2.
\]

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The temperature $T(r, z, \tau)$ in a cooling-off crystal satisfies the following heat conduction equation:

$$\frac{\partial T}{\partial \tau} = \frac{1}{r} \left( \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) \right) + \frac{\partial^2 T}{\partial z^2}, \quad \tau = at, \quad a = \frac{k_s}{c_ps \rho_s},$$

with the boundary conditions

$$-k_s \frac{\partial T}{\partial r} = h_s (T - \Theta_2) \bigg|_{r=R_2}, \quad k_s \frac{\partial T}{\partial r} = h_s (T - \Theta_1) \bigg|_{r=R_1},$$

$$T(r, 0, \tau) = T_m, \quad T(r, l, \tau) = T_c, \quad R_1 \leq r \leq R_2,$$

and the initial condition

$$T(r, z, 0) = T^0(r, z).$$

It is assumed that the temperatures $\Theta_1$, $\Theta_2$ and $T_m$, $T_c$ vary in time according to the exponential law

$$\Theta_1 = T_1^0 e^{-a_1 t} + \frac{z}{l} (T_1^0 e^{-a_2 t} - T_1^0 e^{-a_1 t}),$$

$$\Theta_2 = T_3^0 e^{-a_3 t} + \frac{z}{l} (T_4^0 e^{-a_4 t} - T_3^0 e^{-a_3 t}), \quad T_m = T_m^0 e^{-a_6 t}, \quad T_c = T_c^0 e^{-a_6 t}.$$

We determine the coefficients $a_1$, $a_2$, $\beta_1$, $\beta_2$, $\gamma_1$, and $\gamma_2$ from the condition that we know the final thermal state of the entire system, at which it arrives after a certain known time $t_f$.

We present the solution of problem (4)-(6) in the form of the sum

$$T = T_1 + T^*,$$

where

$$T^* = \lambda \frac{\Theta_1 - \Theta_2}{w_1 + w_2} \ln r + \Theta_1 w_2 + \Theta_2 w_1,$$

$$w_1 = \frac{1}{R_1} - \lambda \ln R_1, \quad w_2 = \frac{1}{R_2} + \lambda \ln R_2, \quad \lambda = \frac{h_s}{k_s}.$$

Let us substitute Eq. (8) into Eqs. (4)-(6) and then apply a Laplace transformation to $T_1$. Then for $\tilde{T}_1$, which is the Laplace transform of the function $T_1$, we obtain the following problem: