A method of successive approximations, a generalization of the Il'yushin method of elastic solutions, is proposed for solving problems of the nonlinear theory of elasticity in which the stress-strain relation is given in the form of a time operator Frechet-differentiable in a neighborhood of zero. The nonlinear relaxation kernels are found from the given nonlinear creep kernels for the principal quadratic theory of elasticity. These relations make it possible to formulate the boundary value problem for this theory. By way of illustration the problem of the pressure exerted on a space by a sphere is examined within the framework of the developed theory. The question of the convergence of the method is discussed in relation to the quadratic theory of visco-elasticity.

1. The term viscoelastic is usually applied to a medium in which the stress-strain relation is given in time operator form. Let the relation take the form:

\[ \sigma = F(e), \]

where \( F \) is a Frechet-differentiable operator, and the abstract functions \( e(\tau) \) and \( \sigma(\tau) \), representing the small-strain tensor and the stress tensor, respectively, are defined on the interval \([0, t]\) and belong to certain Hilbert spaces [1]. If there are no initial stresses, the operator \( F \) can be represented in the form

\[ F(e) = F_0(e) + B(e), \]

where \( \|B(e)\| \to 0 \) as \( \|e\| \to 0 \), and \( F_0(e) \) is an operator linear in \( e \) and therefore representable in integral form. If it is invariant relative to the reference time, its kernel will be a kernel of difference type.

We will consider quasistatic problems. The equilibrium equations take the form [2]

\[ \text{div} \sigma + \rho F = 0, \]

and the strain tensor is related with the displacement vector \( u \) by the Cauchy relations [3]

\[ e = \text{def} u. \]

Substituting in Eqs. (3) the value of the operator \( F \) (1) from Eq. (2) and using relations (4), we obtain the equilibrium equations in the form

\[ \text{div} F_0(\text{def} u) = -\rho F - \text{div} B_1(\text{def} u). \]

Moreover, let the boundary conditions be given; generally speaking, these are of the contact type:

\[ \alpha^{(\alpha)} F(\text{def} u)n + \beta^{(\alpha)} u \frac{E^{(\alpha)}}{l} = N^{(\alpha)}, \]

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where $E^{(n)}$ is the Young's modulus of the elastic contacting body; $\alpha^{(n)}$, $\beta^{(n)}$ are numerical matrices; $l$ is a characteristic linear dimension; $N^{(n)}$ are the so-called "contact forces"; and $\chi$ is the area index, the surface $\Sigma$ bounding the body in question consisting of the sum of all such $\Sigma^{(n)}$. In particular, if $\chi$ takes the values 1 and 2 and the surface forces $S^0$ are given on $\Sigma^{(1)}$ and the displacements $u^0$ on $\Sigma^{(2)}$, then in relations (6)

$$
\alpha^{(1)} = I; \quad \beta^{(1)} = 0; \quad N^{(1)} = S^0; \\
\alpha^{(2)} = 0; \quad \beta^{(2)} = I; \quad N^{(2)} = \frac{E^{(2)}}{E^{(2)}} - u^0,
$$

where $I$ is the unit matrix.

The method of "elastic solutions" consists in the following. We set $B_1 = 0$. We then obtain a linear problem that can be solved, for example, by reduction to the elastic problem by the Laplace transform method. Let $u_0$ be the solution of the corresponding linear problem. To solve the problem by successive approximations, we introduce "fictitious" body and surface forces [2] in accordance with the scheme

$$
\text{div} F_0 = -\rho F = -\rho F - \text{div} B_1 (\text{def} u_0 - \text{def} u_0) = 0;
$$

the boundary conditions

$$
\alpha^{(n)} F_0 (\text{def} u_0) + \beta^{(n)} u - \frac{E^{(n)}}{l} = N^{(n)} = \frac{N^{(n)} - \alpha^{(n)} B_1 (\text{def} u_0)}{l},
$$

being satisfied at each step. These relations are fairly general in character and are valid for a nonlinear medium with arbitrary anisotropy.

Below we consider certain particular cases of representation of the operator $F$.

2. For an isotropic viscoelastic medium the quasilinear stress-strain relations [4] are represented in the form [5]

$$
s = 2 \sum_{n=1}^{\infty} \sum_{k=0}^{n} \hat{G}_{n,k+1}(k^{} \hat{\sigma}^{n-2k+1} \hat{\varepsilon}^{n-2k+1} \hat{\varepsilon});
$$

$$
\hat{\varepsilon} = \sum_{n=1}^{\infty} \sum_{k=0}^{n} \hat{G}_{n,k+1}((n-2k+1) \hat{\sigma}^{n-2k+1} \hat{\varepsilon}^{n-2k+1} \hat{\varepsilon});
$$

and conversely,

$$
e = 2 \sum_{n=1}^{\infty} \sum_{k=0}^{n} \hat{G}_{n,k+1}(k^{} \hat{\sigma}^{n-2k+1} \hat{\varepsilon}^{n-2k+1} \hat{\varepsilon});
$$

$$
\hat{\varepsilon} = \sum_{n=1}^{\infty} \sum_{k=0}^{n} \hat{G}_{n,k+1}((n-2k+1) \hat{\sigma}^{n-2k+1} \hat{\varepsilon}^{n-2k+1} \hat{\varepsilon});
$$

where

$$
\sigma = s + \frac{1}{3} \hat{\sigma}^{n}; \quad \varepsilon = e + \frac{1}{3} \hat{\varepsilon}^{n};
$$

$$
s(t_1) \cdot s(t_2) = \rho (t_1, t_2); \quad e(t_1) \cdot e(t_2) = z(t_1, t_2);
$$

$\hat{G}_{nk}$ and $\hat{G}_{nk}$ are reciprocal integral operators of order $n$; $\nu = \left[ \frac{n+1}{2} \right]$, $\mu = \left[ \frac{n}{2} \right]$ are greatest-integer functions of the arguments.

*Reference [5] contains an error. Equations (11) and (12) of [5] should take the same form as Eqs. (10) and (11) of the present article, respectively.