The axisymmetric two-dimensional flow of a polymer melt in the plane gap of a disk extruder produced by the normal stress effect is considered. The polymer is assumed to be a nonlinear viscoelastic medium, whose strain history is expressed by means of kinematic matrices. A rheological equation of state of the medium, in which all the invariants of the kinematic matrices are functions of strain rate intensity, is established. The laws of distribution of the radial and tangential velocity components over the gap are found from the solution of the equations of motion, and expressions are obtained for the radial pressure distribution and the integral thrust.

Disk extruders are now being increasingly used for polymer processing [1-6]. These machines are characterized by a short residence time in the processing zone, better mixing and homogenization of the polymer than in screw extruders, and simplicity of design.

Several attempts at a theoretical analysis of the disk extruder have been published [2, 4, 7-10]. Most authors [2, 4, 7, 9] base their conclusions on the assumption that the flow takes place under conditions of pure tangential shear and disregard the existence of a radial flow velocity. Thus, they assume that the polymer flow in the direction of the outlet does not affect the nature of the radial pressure distribution. In [2] it was shown that the results obtained contradict the starting assumption and can be taken only as a rough approximation. The authors of [7, 8] disregarded the elastic properties of the moving medium, but in this formulation the problem is physically meaningless, since these properties determine the motion of the material. In [4] it was erroneously assumed that the Weissenberg effect develops "in the case of small deformations at low shear rates." In fact, the Weissenberg effect is associated with the presence of large elastic deformations. The problem of polymer flow in a disk extruder was most thoroughly investigated in [10], where, however, the radial velocity gradient was not taken into account in the expression for the effective viscosity and an expression for the tangential velocity characteristic of pure shear was substituted in the equations of motion. The law of radial velocity distribution over the gap took the form of a quadratic parabola, which is typical of a Newtonian fluid and fails to reflect the viscoelastic behavior of the material. Thus, previous studies of polymer flow in a disk extruder either fail to take the rheodynamics of the process fully into account or are based on incorrect assumptions.

In a disk extruder the polymer melt moves between two flat disks of radius R, one of which rotates about the vertical axis with angular velocity \( \omega \), while the other is stationary. The disks are a distance 2h apart. The fixed cylindrical coordinate system is shown in Fig. 1. We assume that the flow is steady and axisymmetric and that the medium is incompressible and satisfies the continuity equation

\[
\frac{\partial \nu_r}{\partial r} + \frac{\nu_r}{r} + \frac{\partial \nu_z}{\partial z} = 0.
\]  

(1)

We also assume that \( 2h/R = \varepsilon \ll 1 \); then the order of the velocity components is determined from (1) in the form \( v_r \sim v_\phi \); \( v_z \sim v_r \). Hence...
\[ v_r = v_r(r, z); \quad v_\theta = v_\theta(r, z); \quad v_z = 0; \quad \frac{\partial v_r}{\partial r} \ll \frac{\partial v_\theta}{\partial z}; \quad \frac{\partial v_\theta}{\partial r} \ll \frac{\partial v_\theta}{\partial z}. \]  

We shall treat the polymer melt as a nonlinear viscoelastic medium isotropic when at rest. We shall establish a rheological equation of state of the melt, in which the stress tensor components depend on the gradients of the velocities, the accelerations and the higher time derivatives of the velocity expressed in terms of the kinematic matrices \( B_1 = \| B_1 \| \) [11]. In an arbitrary coordinate system the elements of the kinematic matrices take the form \( B^{(1)} = v_m^* g_m^* + v_m^* g_m^* - 2 v_m^* g_m^*; \) \( B^{(n+1)} = \frac{D^{(n)}}{D^t} B^{(n)} + 2 v_m^* B^{(n)} - v_m^* B^{(n)} - v_m^* B^{(n)} \), where \( v_m^* \) is the covariant derivative of \( v_l \) with respect to the coordinate \( x_m^* \); \( g_{ij} \) is the associated metric tensor, and the commas denote covariant differentiation with respect to the coordinate \( x_m^* \). The summation convention applies. The matrix \( B^{(1)} \) for an incompressible medium is equivalent to the classical strain rate tensor for a Newtonian fluid.

Dependences of the stress tensor on the kinematic matrices were obtained in [12, 13] for certain types of flow. For our case in cylindrical coordinates we have

\[ B_1 = \begin{bmatrix} 0 & 0 & \frac{\partial v_r}{\partial z} \\ 0 & 0 & \frac{\partial v_\theta}{\partial z} \\ \frac{\partial v_r}{\partial z} & \frac{\partial v_\theta}{\partial z} & 0 \end{bmatrix}; \]

\[ B_2 = \begin{bmatrix} -2 \left( \frac{\partial v_r}{\partial z} \right)^2 & -2 \frac{\partial v_r}{\partial z} \frac{\partial v_\theta}{\partial z} & 0 \\ -2 \frac{\partial v_r}{\partial z} \frac{\partial v_\theta}{\partial z} & -2 \left( \frac{\partial v_\theta}{\partial z} \right)^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}; \]

\[ B_n = 0 \quad at \quad n > 2. \]

The rheological equation of state is given by the expression \( \sigma = -p I + F(B_1, B_2) \), where \( p \) is hydrostatic pressure; \( I \) is a unit matrix; \( F \) is a matrix whose elements are functions of the elements of \( B_1 \) and \( B_2 \). We assume that the matrix \( F \) is polynomial in the matrices \( B_1 \) and \( B_2 \); then, using the theorem on the reduction of a matrix polynomial to canonical form [14], we obtain

\[ \sigma = -p I + \varphi_1 B_1 + \varphi_2 B_2 + \varphi_3 B_3 + \varphi_4 B_4 + \varphi_5 (B_1 B_2 + B_2 B_1) \]

\[ + \varphi_6 (B_1 B_3 + B_3 B_1) + \varphi_7 (B_2 B_4 + B_4 B_2) + \varphi_8 (B_2^2 B_2 + B_2 B_2^2), \]

where \( \varphi_1 \) are functions of the invariants of \( B_1 \) and \( B_2 \) and their coinvariants. It is easy to see that for the flow in the gap of a disk extruder all these invariants are expressed in terms of the strain rate intensity \( \dot{\epsilon}_1 \), which for the flow in question takes the form

\[ \dot{\epsilon}_1^2 = \left( \frac{\partial v_r}{\partial z} \right)^2 + \left( \frac{\partial v_\theta}{\partial z} \right)^2. \]

Substituting (3) and (4) in (5), we find that the matrix polynomial (5) is equivalent to the expression

\[ \sigma = -p I + \mu_\alpha B_1 - \frac{1}{2} \beta_1 B_2 + \beta_2 \left( B_1^2 + \frac{1}{2} B_2 \right). \]

In this equation the coefficient \( \mu_\alpha \) is the effective viscosity and, if it is assumed that the equation of stationary viscometric flow is described by the Ostwald-deWaele power law, can be written in the form

\[ \mu_\alpha = K \dot{\epsilon}_1^{n-1}, \]

where \( K \) is a constant, and \( n \) is the flow behavior index. \( K \) and \( n \) are found with the aid of a capillary or rotational viscometer. The coefficient \( \beta_1 \) determines the value of the normal stresses on area elements perpendicular to the streamlines and, as shown in [9, 12, 15], is proportional to the square of the viscosity

\[ \beta_1 = A \mu_\alpha^2 = A K \dot{\epsilon}_1^{2(n-1)}, \]