Elastic Plastic Stability of Glass-Reinforced Plastic Plates with a Central Metal Layer

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The problem of the stability of a three-layer plate with a central plastic layer of metal sandwiched between elastic glass-reinforced plastic outer layers is considered. The presence of a metal layer restrains the development of creep strains in the glass-reinforced plastic and makes it possible to neglect the viscous strain components. The general equations of the problem are obtained, and the approximate Il'yushin formulation [1] is considered. An example is presented for a rectangular plate in pure shear. It is shown that the elastic anisotropic layers play the part of a load-relieving system for the central plastic layer [3], which results in an increase in the over-all critical load for the layered plate.

1. We will consider a rigid three-layer plate with elastic orthotropic glass-reinforced plastic outer layers of identical thickness t and a central metal layer of thickness h functioning beyond the elastic limit. To a considerable extent this arrangement inhibits the development of creep strains in the glass-reinforced plastic (GRP). We assume that for the plate as a whole the Kirchhoff hypotheses are satisfied, so that the variations of the strains

\[ \delta \epsilon_{11} = \delta \epsilon_{11} - \tau \delta \kappa_{11}; \quad \delta \epsilon_{22} = \delta \epsilon_{22} - \tau \delta \kappa_{22}; \quad \delta \epsilon_{12} = 2(\delta \epsilon_{12} - \tau \delta \kappa_{12}), \]  

(1.1)

where

\[ \delta \kappa_{11} = \frac{\partial \delta u}{\partial x}; \quad \delta \kappa_{22} = \frac{\partial \delta v}{\partial y}; \quad \delta \kappa_{12} = \frac{\partial \delta w}{\partial x \partial y}; \]  

\[ \delta \kappa_{11} = \frac{\partial^2 \delta w}{\partial x^2}; \quad \delta \kappa_{22} = \frac{\partial^2 \delta w}{\partial y^2}; \quad \delta \kappa_{12} = \frac{\partial \delta w}{\partial x \partial y}; \]  

(1.2)

\( \delta u, \delta v, \delta w \) are the variations of the displacements associated with buckling.

In solving the stability problem for plates the following equations must be satisfied [1]

\[ \frac{\partial \delta T_{11}}{\partial x} + \frac{\partial \delta T_{12}}{\partial y} = 0; \quad \frac{\partial \delta T_{21}}{\partial x} + \frac{\partial \delta T_{22}}{\partial y} = 0; \]  

(1.3)

\[ \frac{\partial \delta M_{11}}{\partial x^2} + 2 \frac{\partial \delta M_{12}}{\partial x \partial y} + \frac{\partial \delta M_{22}}{\partial y^2} + T_{11} \delta \kappa_{11} + T_{22} \delta \kappa_{22} + 2 T_{12} \delta \kappa_{12} = 0; \]  

(1.4)

\[ \frac{\partial^2 \delta \kappa_{11}}{\partial y^2} + \frac{\partial^2 \delta \kappa_{22}}{\partial x^2} - 2 \frac{\partial \delta \kappa_{12}}{\partial x \partial y} = 0, \]  

(1.5)

where \( \delta T_{ij} \) are the additional forces, and \( \delta M_{ij} \) the additional moments associated with buckling of the plate. We represent the forces in the plate before buckling in the form \( T_{ij} = T'_{ij} + T''_{ij} + T'''_{ij} \), where the single and triple primes relate to the outer layers on the convex and concave sides, respectively, and double prime to the central layer; \( T'_{ij} = T''_{ij} \). Buckling of the plate results in a redistribution of forces among the layers, so that

\[ \delta T_{ij} = \delta T'_{ij} + \delta T''_{ij} + \delta T'''_{ij}. \]  

(1.6)
The expressions for the variations of the forces and moments in each layer can be represented in the form [1, 2]

\[
\frac{\delta T'_{11}}{E_x} = \frac{h+t}{4} \delta \varepsilon_{11} + \mu_{xy} \frac{h+t}{2} \delta \chi_{11} + \frac{h+t}{2} \delta \varepsilon_{22} - \frac{h+t}{2} \delta \chi_{22};
\]

\[
\frac{\delta T'_{12}}{2G_{xy}} = \frac{h+t}{4} \delta \varepsilon_{12} - \frac{h+t}{2} \delta \chi_{12} ;
\]

\[
\frac{\delta T''_{11}}{E_x} = \frac{h+t}{4} \delta \varepsilon_{11} + \mu_{xy} \frac{h+t}{2} \delta \chi_{11} + \frac{h+t}{2} \delta \varepsilon_{22} - \frac{h+t}{2} \delta \chi_{22} ;
\]

\[
\frac{\delta T''_{12}}{2G_{xy}} = \frac{h+t}{4} \delta \varepsilon_{12} - \frac{h+t}{2} \delta \chi_{12} ;
\]

\[
\frac{\delta T''_{11}}{E_x} = \frac{4E_x}{3} (h - \Omega_1) \left( \delta \varepsilon_{11} + \frac{1}{2} \delta \varepsilon_{22} \right) + \frac{4E_x}{3} \Omega_2 \left( \delta \chi_{11} + \frac{1}{2} \delta \chi_{22} \right) - \frac{T''_{12}}{h(1 - \omega)}
\]

\[
\frac{\delta T''_{12}}{2E_x} = \frac{2E_x}{3} (h - \Omega_1) \delta \varepsilon_{12} + \frac{2E_x}{3} \Omega_2 \delta \chi_{12} - \frac{T''_{12}}{h(1 - \omega)}
\]

\[
\delta M'_{11} = -D_x (\delta \chi_{11} + \mu_{xy} \delta \varepsilon_{22}) + \frac{E_x S'}{1 - \mu_{xy} \mu_{xz}} \delta \varepsilon_{11} + \mu_{xy} \delta \varepsilon_{22} ;
\]

\[
\delta M''_{11} = -D_x (\delta \chi_{11} + \mu_{xy} \delta \varepsilon_{22}) + \frac{E_x S''}{1 - \mu_{xy} \mu_{xz}} \delta \varepsilon_{11} + \mu_{xy} \delta \varepsilon_{22} ;
\]

\[
\delta M'_{22} = -D_y (\delta \chi_{22} + \mu_{xy} \delta \varepsilon_{11}) + \frac{E_y S'}{1 - \mu_{xy} \mu_{yz}} \delta \varepsilon_{11} + \mu_{xy} \delta \varepsilon_{22} ;
\]

\[
\delta M''_{22} = -D_y (\delta \chi_{22} + \mu_{xy} \delta \varepsilon_{11}) + \frac{E_y S''}{1 - \mu_{xy} \mu_{yz}} \delta \varepsilon_{11} + \mu_{xy} \delta \varepsilon_{22} ;
\]

\[
\delta M'_{12} = -2D_x \delta \chi_{12} + 2G_{xy} S' \delta \varepsilon_{12} ;
\]

\[
\delta M''_{12} = -2D_x \delta \chi_{12} + 2G_{xy} S'' \delta \varepsilon_{12} ;
\]

Here,

\[
\Omega_1 = \frac{\omega h}{2} \left( 1 - z^* \right) ; \quad \Omega_2 = \frac{\omega h}{8} \left( 1 - z^* \right) ; \quad \Omega_3 = \frac{\omega h}{24} \left( 1 - z^* \right) ;
\]

\[
\delta \Omega_1 = -\frac{(\lambda - \omega) h^2}{8e''_{ii}} (1 - z^*)^2 \delta \chi ; \quad \delta \Omega_2 = -\frac{(\lambda - \omega) h^2}{48e''_{ii}} (2 + z^*) (1 - z^*) (1 - z^*)^2 \delta \chi
\]

for the region of elastoplastic strains;

\[
\Omega_1 = \omega h ; \quad \Omega_2 = 0 ; \quad \Omega_3 = \frac{\omega h^3}{12} ; \quad \delta \Omega_1 = \frac{(\lambda - \omega) h^3}{e''_{ii}} \delta \varepsilon ;
\]

\[
\delta \Omega_2 = -\frac{(\lambda - \omega) h^3}{12e''_{ii}} \delta \chi
\]

for the region of active plastic strains, and

\[
\Omega_1 = \Omega_2 = \Omega_3 = 0 ; \quad \delta \Omega_1 = \delta \Omega_2 = 0
\]

for the region of unloading of the central layer;

\[
\delta e = \tilde{\sigma}'_{11} \delta \varepsilon_{11} + \tilde{\sigma}'_{22} \delta \varepsilon_{22} + 2\tilde{\sigma}'_{12} \delta \varepsilon_{12} ; \quad \delta \chi = \tilde{\sigma}'_{11} \delta \chi_{11} + \tilde{\sigma}'_{22} \delta \chi_{22} + \tilde{\sigma}'_{12} \delta \chi_{12} ;
\]

\[
\delta e'' = \delta e - z_0 \delta \chi = 0 ;
\]

\[
E_x, E_y \text{ are the moduli of elasticity of GRP in the principal directions of anisotropy; } \mu_{xy}, \mu_{yz} \text{ are the Poisson's ratios; } E_z \text{ is the modulus of elasticity; } \lambda \text{ and } \omega \text{ the plasticity parameters for the central layer; } z_0 = 2z_0/h \text{ is the dimensionless boundary of the unloading zone; } \sigma''_{ij}, \varepsilon''_{ij} \text{ are the stress and strain intensities; } \tilde{\sigma}_{ij} = \sigma''_{ij}/\sigma''_{i} ; \quad S'' = -S'' = 0.5t(1 + h) ;
\]

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