ANALYSIS OF THE STRENGTH OF GLASS-FIBER-REINFORCED SHELLS UPON LOSS OF STABILITY

Ya. A. Brauns and R. B. Rikards

An algorithm is developed for calculating and analyzing the components of the stress tensor on the basis of an experimental function of deflections in the case of buckling of glass-fiber-reinforced shells loaded by a hydrostatic load. Possible sites of fracture of a shell are established qualitatively with the use of Malmeister's strength theory.

An investigation of the forms of buckling of circular cylindrical shells has been the subject of many experimental and theoretical studies [1, 2]. On the basis of analyzing the forms of equilibrium states it is possible to determine the stability of a shell on the whole, especially if a statistical analysis is used [1]. It is also known that the form of buckling determines the state of stress of the shell. The purpose of this work is to analyze the stressed state in the case of short-time loading of orthotropic glass-fiber-reinforced shells by a hydrostatic load. The radial displacement field for a series of shells during loading was investigated in [3]. The displacement function lends itself to a determinate description and is characterized by the sum of the harmonics of a Fourier series. In the present work the averaged stresses are calculated with the use of a set of coefficients of the series, and the buckling strength of shells of fabric polyester glass-fiber-reinforced plastic [fabric ASTT(b)-C4-O, resin PN-3] is analyzed.

1. Let us consider the cylindrical shell of orthotropic glass-fiber-reinforced plastic. We denote the length of the shell by \(L\), the radius \(R\), and the thickness \(h\); we direct axes \(x, y, z\) along the generatrix, along the circumference, and along the normal to the center of curvature; these axes coincide with the axes of symmetry of the material. Using the Kirchhoff-Love hypothesis, we determine the total strains and stresses as the sum of the strains and stresses in the middle surface and of the bending strains and stresses in the form

\[
\varepsilon_{\alpha\beta}(x, y, z) = \varepsilon_{\alpha\beta}^0(x, y) + z \kappa_{\alpha\beta}(x, y) - \kappa_{\alpha\beta}^0(x, y);
\]

\[
\sigma_{\alpha\beta}(x, y, z) = \sigma_{\alpha\beta}^0(x, y) + \sigma_{\alpha\beta}^1(x, y) - \sigma_{\alpha\beta}^0(x, y);
\]

Here \(\varepsilon_{\alpha\beta}^0, \sigma_{\alpha\beta}^0\) are membrane strain and stress, respectively; \(\kappa_{\alpha\beta}\) is the distortional tensor; \(A_{\alpha\beta\gamma\delta}\) are the stiffness components of the material. In the case of the Kirchhoff-Love kinematic model in which it is assumed that the normal to the middle surface remains normal also after deformation, the stress function of the middle surface of the shell \(\Phi(x, y)\) is related uniquely with the displacement function \(w(x, y)\) by the equation of strain compatibility. In determining the stress function we will use the equation of strain compatibility for cylindrical shells of orthotropic material:

\[
\frac{1}{R} \frac{\partial^2 w(x, y) - w_0(x, y)}{\partial x^2} = - \frac{\partial^2 \Phi(x, y)}{\partial x^2 \partial y^2},
\]

where \( a_{ijkl} \) are the compliance components of the material.

We solve differential equation (3) by expressing the known function of deflections \( w(x, y) \), \( w_0(x, y) \) by trigonometric functions

\[
w(x, y) = \sum_{m=0}^{M} \sum_{n=0}^{N} a_{mn} \cos \frac{m \pi x}{L} \cos \frac{n \pi y}{R} + \sum_{m=0}^{M} \sum_{n=1}^{N} b_{mn} \cos \frac{m \pi x}{L} \sin \frac{n \pi y}{R} + \sum_{m=1}^{M} \sum_{n=0}^{N} c_{mn} \sin \frac{m \pi x}{L} \cos \frac{n \pi y}{R} + \sum_{m=1}^{M} \sum_{n=1}^{N} d_{mn} \sin \frac{m \pi x}{L} \sin \frac{n \pi y}{R}; \tag{4}
\]

\[
\omega_0(x, y) = \sum_{m=0}^{M} \sum_{n=0}^{N} a_{mn} \cos \frac{m \pi x}{L} \cos \frac{n \pi y}{R} + \sum_{m=0}^{M} \sum_{n=1}^{N} b_{mn} \cos \frac{m \pi x}{L} \sin \frac{n \pi y}{R} + \sum_{m=1}^{M} \sum_{n=0}^{N} c_{mn} \sin \frac{m \pi x}{L} \cos \frac{n \pi y}{R} + \sum_{m=1}^{M} \sum_{n=1}^{N} d_{mn} \sin \frac{m \pi x}{L} \sin \frac{n \pi y}{R}; \tag{5}
\]

In the expression for the stress function we take into account the load from pressure \( q \) on the side and end surfaces of the shell:

\[
\Phi(x, y) = \sum_{m=0}^{M} \sum_{n=0}^{N} \alpha_{mn} \cos \frac{m \pi x}{L} \cos \frac{n \pi y}{R} + \sum_{m=0}^{M} \sum_{n=1}^{N} \beta_{mn} \cos \frac{m \pi x}{L} \sin \frac{n \pi y}{R} + \sum_{m=1}^{M} \sum_{n=0}^{N} \gamma_{mn} \sin \frac{m \pi x}{L} \cos \frac{n \pi y}{R} + \sum_{m=1}^{M} \sum_{n=1}^{N} \delta_{mn} \sin \frac{m \pi x}{L} \sin \frac{n \pi y}{R} + qRy^2 \tag{6}
\]

Substituting expressions (4)-(6) into Eq. (3), we establish the relation between coefficients of the stress function of the middle surface and the deflection function. This relation is expressed by

\[
[X_{mn}] = - \frac{1}{A_{mn}} ([B_{mn}] - [B_{omn}]), \tag{7}
\]

where

\[
[X_{mn}] = \begin{bmatrix} \alpha_{mn} \\
\beta_{mn} \\
\gamma_{mn} \\
\delta_{mn} \end{bmatrix}; \quad [B_{mn}] = \begin{bmatrix} a_{mn} \\
b_{mn} \\
c_{mn} \\
d_{mn} \end{bmatrix}; \quad [B_{omn}] = \begin{bmatrix} a_{omn} \\
b_{omn} \\
c_{omn} \\
d_{omn} \end{bmatrix}; \tag{8}
\]

the coefficients \( A_{mn} = a_{111n}L^4 + \frac{\pi^2m^2R^2}{L^2} + a_{2222n^2}R^2 \),\( R^2 + (2a_{1122} + 4a_{1212})n^2/R \). Here \( m \) is the number of half-waves along the generatrix; \( n \) is the number of waves along the circumference.

In [3] the third and fourth groups of terms of the expansion were used for approximation of the deflection function, which gives a good representation of the experimental function. According to the experimental data obtained in [3], we take the maximum number of harmonics of the series along the generatrix \( M = 4 \) and along the circumference \( N = 6 \).

The final expressions for stresses in the middle surface are

\[
\sigma_{11}(x, y) = \frac{\partial^2 \Phi(x, y)}{\partial y^2} = \frac{1}{R^2} \left[ \sum_{m=1}^{4} \sum_{n=0}^{6} \gamma_{mn} n^2 \lambda (x, y) + \sum_{m=1}^{4} \sum_{n=1}^{6} \delta_{mn} n^2 \xi (x, y) \right] - \frac{qR_2}{2h}; \tag{9}
\]

\[
\sigma_{22}(x, y) = \frac{\partial^2 \Phi(x, y)}{\partial x^2} = \frac{\pi^2}{L^2} \left[ \sum_{m=1}^{4} \sum_{n=0}^{6} \gamma_{mn} m^2 \lambda (x, y) + \sum_{m=1}^{4} \sum_{n=1}^{6} \delta_{mn} m^2 \xi (x, y) \right] - \frac{qR_1}{h}; \tag{10}
\]