EFFECT OF THE ARRANGEMENT OF THE REINFORCEMENT ON THE STABILITY OF CIRCULAR, CYLINDRICALLY ORTHOTROPIC PLATES WITH LOW SHEAR STRENGTH

V. V. Khitrov

The Bubnov-Galerkin method is used to find the critical loads for cylindrically orthotropic circular continuous and annular plates with different edge conditions in axisymmetric buckling. The effect of the ratio of the Young moduli in the circumferential \( E_\theta \) and radial \( E_r \) directions and the ratio of \( E_\theta \) to the interlaminar shear modulus \( G_{rz} \) is investigated. The results are in qualitative agreement with the experimental data of [13].

The carrying capacity of fiber-reinforced plates is often determined by the stability. Accordingly, the problem of how to arrange the reinforcement so as to obtain maximum stiffness is of primary practical importance. However, existing solutions either disregard the anisotropy in the plane of the plate [1-4] (the case \( E_\theta = E_r \) is considered) or are obtained on the basis of the Kirchhoff-Love hypothesis [5-7], which, as shown in [8], may lead to serious errors when applied to fiber-reinforced materials. The results of [9] can be used only when \( E_\theta \) is close to \( E_r \), since the nonuniformity of the state of stress was not taken into account.

A circular plate with outside radius \( R_o \), inside radius \( R_i \), and thickness \( 2H \), considered in the cylindrical coordinate system \( r, \theta, z \), is subjected to an axisymmetric load \( T \) acting in the middle surface, which coincides with the plane \( r \theta \).

We characterize the cylindrical orthotropy, determined by the arrangement of the material in the radial and circumferential directions, and the low shear strength by means of the anisotropy parameters

\[ \alpha = \sqrt{\frac{E_\theta}{E_r}} \quad \text{and} \quad \kappa = \frac{2H}{R_o} \sqrt{\frac{E_\theta}{G_{rz}}} \]

We make the following assumptions:

1) there is no transverse strain: \( \xi_z = 0 \) [8];

2) \( E_r \) and \( E_\theta \) are constants, which can be realized by taking into account the structural nature of the anisotropy [5], \( G_{rz} \) is a constant that does not depend on \( \alpha \), being chiefly determined by the characteristics of the resin and the resin content;

3) buckling is axisymmetric.

For isotropic plates depending on the value of \( k = R_i/R_o \) and the conditions at the supports, the critical loads obtained with the latter assumption may be too high, since, in fact, a smaller critical load corresponding to buckling with the formation of nodal diameters is realized [4]. The question of the buckled shapes of orthotropic plates is still in the investigation stage [10].

The stability equations for the axisymmetric deformation of circular plates have the form [8]

\[
\frac{\partial \sigma_r}{\partial \rho} + \frac{\sigma_r - \sigma_\theta}{\rho} + R_o \frac{\partial \tau_{rz}}{\partial z} = 0; \\
\frac{dM_1}{d\rho} + \frac{M_1 - M_2}{\rho} + N_1 \frac{d\psi}{d\rho} = 0. \quad (1,2)
\]
The normal stresses in the radial and circumferential directions $\sigma_r$ and $\sigma_\theta$ and the shear stress $\tau_{rz}$ are given by [8]

$$
\sigma_r = \frac{E_r}{1-v^2} \left( \frac{\partial u}{\partial \rho} + \frac{v}{\rho} \frac{u}{\rho} \right); \quad \sigma_\theta = \frac{E_\theta}{1-v^2} \left( \frac{u}{\rho} + \frac{v}{\alpha^2} \frac{\partial u}{\partial \rho} \right); \quad \tau_{rz} = G \left( \frac{1}{R_o} \frac{\partial w}{\partial \rho} + \frac{\partial u}{\partial z} \right),
$$

where $E_r = \frac{E_r}{1-v^2}$; $E_\theta = \frac{E_\theta}{1-v^2}$; $v = v_0 = v c^2$ is Poisson’s ratio characterizing the shortening in the $r$ direction associated with stretching in the $\theta$ direction; $u$ and $w$ are the displacements along the axes $r$ and $z$; $\rho = r/R_o$ is the nondimensional radius; $M_1$, $M_2$, and $N_1$ are the bending moments in the radial and circumferential directions and the radial force given by the expressions [8]

$$
M_1 = \int_{-H}^{H} z \sigma_r dz; \quad M_2 = \int_{-H}^{H} z \sigma_\theta dz; \quad N_1 = \int_{-H}^{H} \sigma_r dz.
$$

Using (3), we rewrite Eq. (1) in displacements

$$
\frac{\partial^2 u}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial u}{\partial \rho} - \alpha^2 \frac{u}{\rho^2} + \frac{R_o^2}{\beta^2} \frac{\partial^2 u}{\partial z^2} = 0,
$$

where $\beta^2 = \frac{E_r}{G}$. The solution of Eq. (5) can be obtained by the Fourier method and is represented in the form [9]

$$
u = A l_\alpha (\lambda \rho) \sin \frac{\beta}{R_o} z + A_1 l_\alpha (\lambda \rho) \cos \left( \frac{\beta}{R_o} z + A_2 Y_\alpha (\lambda \rho) \sin \left( \frac{\beta}{R_o} z + A_3 Y_\alpha (\lambda \rho) \cos \right) \right)
$$

where $A$ and $B$ are constants; $l_\alpha$ and $Y_\alpha$ are Bessel functions of the first and second kinds of order $\alpha$; $\lambda$ is a constant, which, as shown in [3, 9], characterizes the critical buckling load.

Since there cannot be several critical loads corresponding to the same buckling mode, it is necessary to set $\lambda_1 = \lambda_2 = \lambda_3 = \lambda$. 

![Fig. 1. Square of the critical load parameter as a function of the anisotropy parameter for cylindrically orthotropic shear-compliant continuous plates. $k = 0$; $\nu = 0.3$; $\lambda = 3.83$ (a) and 2.05 (b). $\nu_0 = 0$ (1); 0.2 (2); 0.4 (3); 0.6 (4), 0.8 (5); 1.0 (6); 1.2 (7).](image)