The coupling equations of hard cellular polymers and glass plastics based on these are analyzed. The material is considered as an elastically relaxing medium for the case of small deformations. The physical relationship between the rubber-elastic stresses and strains is derived in explicit form for the case of attenuating creep and a uniform three-dimensional stressed state. The total deformations are described by the one-dimensional rheological model of a "typical body" with a reduced stress and an instantaneous elastic modulus, having the appropriately chosen viscosity of a non-Newtonian liquid.

This analysis constitutes the development of an approach first presented in an earlier paper [1] in relation to the three-dimensional stressed state. We shall consider hard (rigid) cellular polymers and glass plastics based on these materials. It has been found experimentally that, when studying the deformation characteristics of these, attention may be confined to elastic and rubber-elastic deformations which are small in comparison with unity, and residual deformations being entirely excluded. Thus the material is regarded as an elastically relaxing medium in a small-deformation range, i.e.,

$$\varepsilon = \varepsilon^s + \varepsilon^0 = \varepsilon + \varepsilon^s \quad (\varepsilon^0 = 0),$$

where $\varepsilon$ is the total deformation, $\varepsilon^s$ is the rubber-elastic (reversible, inelastic in the conventional sense) deformation constituting the result of a reversible change in the conformation of the high-molecular-weight structures and their complexes, $\varepsilon^0$ is the residual (irreversible inelastic) deformation constituting the result of an irreversible regrouping of the particles, and $\varepsilon$ is the (conventional) elastic deformation due to small changes in the interatomic or intermolecular distances.

We shall regard the material as homogeneous and isotropic. The stressed state will be considered uniform and the deformation process isothermal. A detached prismatic element of the continuous medium loaded with the principal stresses is shown in Fig. 1. The stresses $\sigma_i$ ($i = 1, 2, 3$) are applied simultaneously and instantaneously (we neglect inertial forces). We thus have an equation $\sigma_i = \text{const}$ with $t = 0$ to $t = \infty$. The elastic strains are, according to Hooke's law,

$$e_i = \frac{\sigma_i^s}{E}, \quad e_2 = \frac{\sigma_2^s}{E}, \quad e_3 = \frac{\sigma_3^s}{E},$$

where

$$\sigma_1^s = \sigma_1 - \mu (\sigma_2 + \sigma_3); \quad \sigma_2^s = \sigma_2 - \mu (\sigma_1 + \sigma_3); \quad \sigma_3^s = \sigma_3 - \mu (\sigma_1 + \sigma_2);$$

$E$ is the instantaneous elastic modulus, $\mu$ is the Poisson coefficient.

Following [2] we may write the nonlinear physical relationship (the coupling equations) between the stresses and rubber-elastic deformations in the form

$$\dot{\varepsilon}_1^s = \frac{f_1^s}{\eta^s}; \quad \dot{\varepsilon}_2^s = \frac{f_2^s}{\eta^s}; \quad \dot{\varepsilon}_3^s = \frac{f_3^s}{\eta^s}.$$
Here the dot indicates differentiation with respect to time. The functions \( f^*_i \) are given in the form

\[
\begin{aligned}
  f^*_1 &= \tilde{\sigma}_1 - E\varepsilon^*_1; \\
  f^*_2 &= \tilde{\sigma}_2 - E\varepsilon^*_2; \\
  f^*_3 &= \tilde{\sigma}_3 - E\varepsilon^*_3.
\end{aligned}
\]

(5)

where

\[
\begin{aligned}
  \tilde{\sigma}_1 &= \sigma_1 - 0.5(\sigma_2 + \sigma_3); \\
  \tilde{\sigma}_2 &= \sigma_2 - 0.5(\sigma_1 + \sigma_3); \\
  \tilde{\sigma}_3 &= \sigma_3 - 0.5(\sigma_1 + \sigma_2).
\end{aligned}
\]

(6)

\( E_\infty \) is the modulus of rubber-elastic strain. The viscosity \( \eta^* \) equals

\[
\eta^* = \eta^*_0 \exp \left[-\frac{1}{m^*} \left(\gamma^* p + |f^*|_{\max}\right)\right],
\]

(7)

where \( p = (\sigma_1 + \sigma_2 + \sigma_3)/3 \) is the mean stress, \( \eta^*_0 \) is the initial viscosity, \( m^* \) is the logarithmic velocity modulus, \( \gamma^* \) is the volume coefficient.

On the basis of (4) and (5) we obtain the following for the coupling equations:

\[
\tilde{\sigma}_i = E\varepsilon^*_i + \eta^* \varepsilon^*_i \quad (i = 1, 2, 3).
\]

(8)

An exact solution of (8) leads to integral exponentials. The relationship between the stresses and rubber-elastic strains is then obtained in implicit form, and this complicates its practical use.

The aim of the present investigation is to reduce the three-dimensional case to one-dimensional and to select an expression for the viscosity \( \eta^* \) such as will reasonably accurately describe the relationship between the stresses, the rubber-elastic strains, and the time in explicit form.

Equation (8) corresponds to a rheological model with the viscosity of a non-Newtonian liquid (Fig. 2). Hence creep in a triaxial stressed state may be described by the one-dimensional model of a "typical body" with a reduced stress \( \tilde{\sigma}_i \) and a viscosity \( \eta^* \) depending on the stresses in the three principal directions \( \sigma_1, \sigma_2, \sigma_3 \). Depending on the sign of \( \tilde{\sigma}_i \), either tensile or compressive rheological constants, determined by means of uniaxial tests, are introduced into Eq. (8). In Eq. (7), the way in which the quantity \( E_\infty = E'\infty \) participates in \( |f^*_1|_{\max} \) depends on the sign of \( |\tilde{\sigma}_i|_{\max} \). It is always assumed [2] that \( |f^*_1|_{\max} \) and \( |\tilde{\sigma}_i|_{\max} \) max act in the same direction.

The elastic deformation is taken into account by successively attaching a spring with an elastic modulus \( E' = E = \tilde{\sigma}_i / \sigma_i \) to the model (Figs. 2 and 3). Hence the total deformations of an elastically relaxing medium subject to a uniform stressed state may be described by the one-dimensional model of a "typical body" involving the reduced stress \( \tilde{\sigma}_i \), the reduced instantaneous elastic modulus \( E' \), and the viscosity \( \eta^* \).

The time development of the rubber-elastic deformation is shown in Fig. 4 in accordance with the model of Fig. 2 for \( \tilde{\sigma}_i = \text{const} \). The \( \tilde{\sigma}_i \) and \( \varepsilon^*_i \) have identical signs, and since \( |\varepsilon^*_i| \leq \frac{\sigma_i}{E_\infty} \), we have

\[
|f^*|_{\max} = |\tilde{\sigma}_i|_{\max} - |E'\varepsilon^*_i|_{\max},
\]

(9)

where \( \varepsilon^*_i \) is the rubber-elastic deformation corresponding to \( |\tilde{\sigma}_i|_{\max} \).

Allowing for (7) and (9), we have

\[
\eta^* = \eta^*_0 \exp \left[-\frac{|E'\varepsilon^*_i|_{\max} - |\tilde{\sigma}_i|_{\max}}{m^*} - \frac{\gamma^* p}{m^*}\right].
\]

(10)